Long wave propagation and bore dynamics in coastal and estuarine environments

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Sumatra 2004, tsunami reaching the coast of Thailand, Madsen et al. (2008)
non-hydrostatic processes

Tissier, Bonneton et al., JCR 2011

(a) and (b)
Long waves and non-hydrostatic processes

**Tsunamis**

2011 great Tohoku tsunami; Naka river at Hitachinaka city

**Tidal waves**

→ tidal bores

*Tidal bore, Bonneton et al. 2011*
Long waves and non-hydrostatic processes

Nearshore wind waves

- Understanding of non-hydrostatic phenomena and breaking
- Development of efficient long-wave modelling approaches
Introduction

Collaborations

- **long wave modelling**
  - Eric Barthélémy, LEGI, Grenoble
  - Rodrigo Cienfuegos, PUC, Santiago de Chile
  - Marion Tissier, TU Delft
  - David Lannes, ENS, Paris
  - Fabien Marche, I3M, Montpellier
  - Mario Ricchiuto, INRIA, Bordeaux

- **Nha Trang project / MOST** Vietnam / France
  - Nguyen Trung Viet, Water Resources University
  - Dinh Van Uu, Hanoi University of Science
  - Rafael Almar, J-P. Lefebvre, IRD, France
  - Natalie Bonneton, Philippe Bonneton, EPOC, France
Introduction

Observation of non-hydrostatic processes
  → tidal wave propagation and tidal bore formation

Non-hydrostatic modelling
  • Theoretical background
  • A new approach
  • Validations

Conclusion and perspectives
Introduction

Observation of non-hydrostatic processes
→ tidal wave propagation and tidal bore formation

Non-hydrostatic modelling
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Conclusion and perspectives
Large amplitude spring tide – 10th September 2010

Bonneton et al., 2011
**Tidal bore**: a fascinating hydrodynamic phenomenon observed worldwide

Tidal bore occurrence is strongly underestimated

*see Bonneton et al., 2012, 2014*
Observation of non-hydrostatic processes

Tidal waves

\[ \varepsilon = \frac{T_R}{D_1} \]

Garonne River, Bonneton et al., 2014
Observation of non-hydrostatic processes

Tidal waves

\[ \frac{\partial \mathbf{D}}{\partial \mathbf{A}} = -\mathbf{A} \]

Low intensity tidal bores

Garonne River, Bonneton et al., 2014
Observation of non-hydrostatic processes

Tidal waves

F=1.08

F=1.24

low intensity tidal bore

high intensity tidal bore

most of the time this phenomenon is ignored in estuaries
Seine estuary

Field site
100 km from the estuary mouth

Bonneton et al., 2012
need to reassess tidal bore occurrence in meso and macro-tidal estuaries worldwide (including Asian & Pacific estuaries)
→ high frequency measurements are required

tidal bores play a significant role in estuarine ecosystems
Outline

❑ Introduction

❑ Observation of non-hydrostatic processes
  → tidal wave propagation and tidal bore formation

❑ Non-hydrostatic modelling
  • Theoretical background
  • A new approach
  • Validations

❑ Conclusion and perspectives
Non-hydrostatic modelling

Theoretical background

\[ z = \zeta(x,t) \]

\[ \varepsilon = \frac{A}{d} \]

\[ \mu = \left( \frac{d}{\lambda} \right)^2 \]

\[ \mu \leq 0.01 \]

\[ \varepsilon = O(1) \rightarrow \varepsilon = O(\mu) \]
Non-hydrostatic modelling

Theoretical background

\[ \mu = \left( \frac{d_0}{\lambda_0} \right)^2 \ll 1 \]

\[ \varepsilon = \frac{A_0}{d_0} = O(1) \]

Inviscid 3D incompressible irrotational Euler equations

\[ \partial_t \zeta + \nabla \cdot (h \mathbf{u}) = 0 \]

\[ \partial_t \mathbf{u} + \varepsilon (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla \zeta = \mu \mathcal{D} + O(\mu^2) \]

\[ \varepsilon = O(1) \]

Serre or Green Naghdi equations
Non-hydrostatic modelling

\[ \partial_t \zeta + \nabla \cdot (hu) = 0 \]
\[ \partial_t u + \varepsilon (u \cdot \nabla) u + \nabla \zeta = \mu \mathcal{D} + O(\mu^2) \]

Lannes and Bonneton (2009)

\[ \mathcal{D} = -T[h, b]u_t - \varepsilon Q[h, b](u) \]

where the linear operator \( T[h, b] \) is defined as

\[ T[h, b]W = -\frac{1}{3h} \nabla (h^3 \nabla \cdot W) + \frac{1}{2h} \left[ \nabla (h^2 \nabla b \cdot W) - h^2 \nabla b \nabla \cdot W \right] + \nabla b \nabla b \cdot W \]

and the quadratic term \( Q[h, b](u) \) is given by

\[ Q[h, b](u) = -\frac{1}{3h} \nabla \left( h^3 ((u \cdot \nabla) (\nabla \cdot u) - (\nabla \cdot u)^2) \right) \]
\[ + \frac{1}{2h} \left[ \nabla (h^2 (u \cdot \nabla)^2 b) - h^2 ((u \cdot \nabla) (\nabla \cdot u) - (\nabla \cdot u)^2) \nabla b \right] + ((u \cdot \nabla)^2 b) \nabla b \]
Reformulation of SGN equations

\[ \partial_t h + \nabla \cdot (hu) = 0 \]

\[ \partial_t (hu) + \nabla \cdot (hu \otimes u) + \nabla \left( \frac{1}{2}gh^2 \right) = -gh\nabla b \]

\[ + \frac{1}{\alpha}gh\nabla \zeta - \left( I + \alpha h T \frac{1}{h} \right)^{-1} \left[ \frac{1}{\alpha}gh\nabla \zeta + h Q_1(u) \right] \]

\[ Q_1(u) = Q(u) - T((u \cdot \nabla)u) \] only involves second order derivatives of \( u \)

\[ \alpha \rightarrow \text{improved dispersive properties (Madsen et al., 1991)} \]

\[ kd_0 \leq 3 \]

_Bonneton, Chazel, Lannes, Marche and Tissier (2011)_
Lannes and Marche (2014) have proposed a new formulation where the operator to invert is time independent

→ a considerable decrease of the computational time!
Non-hydrostatic modelling

A new approach

Hybrid method

\[
\begin{align*}
\partial_t \zeta + \nabla \cdot (hu) &= 0 \\
\partial_t u + \varepsilon (u \cdot \nabla) u + \nabla \zeta &= \mu \mathcal{D}
\end{align*}
\]

non-breaking waves: SGN

broken wave fronts and swash motions: NSWE
Shoaling and breaking of regular waves over a sloping beach
Shoaling and breaking of regular waves over a sloping beach
Non-hydrostatic modelling

Validations

Shoaling and breaking of regular waves over a sloping beach

Validation with Cox (1995) experiments

Tissier et al., 2012
Non-hydrostatic modelling

Validations

Undular bore propagation

Laboratory experiments by Soares-Frazao et Zech (2002), Fr = 1.104
Non-hydrostatic modelling

Undular bore propagation

Fr = 1.10
Fr = 1.20
Fr = 1.25
Fr = 1.35
Fr = 1.37
Fr = 1.40
Fr = 1.50
Fr = 1.90

$\zeta (h_2 - h_1)$

$x (m)$

$t = 24s$
Wave overtopping and multiple shorelines

BARDEX II - HYDRALAB project

Bonneton et al., 2013
Non-hydrostatic modelling

Validations

Wave overtopping and multiple shorelines

BARDEX II - HYDRALAB project

Bonneton et al., 2013
Wave overtopping and multiple shorelines

Solitary waves overtopping a seawall (Hsiao and Lin, 2010)

Tissier et al., 2012
Wave overtopping and multiple shorelines

Solitary waves overtopping a seawall (Hsiao and Lin, 2010)

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Solitary waves overtopping a seawall (Hsiao and Lin, 2010)

Tissier et al., 2012
Wave overtopping and multiple shorelines

Hsiao et Lin (2010)
COBRAS model
2D VOF model
RANS equations $K-\varepsilon$

Non-hydrostatic modelling

Validations

SURF-GN
Non-hydrostatic modelling

Long wave propagation in the swash zone

Truc Vert Beach 2001
- Offshore wave conditions: $\theta \approx 0^\circ$, $H_s=3$ m, $T_s=12$ s
- Maximum surf zone width: 500 m

Bottom topography and pressure sensor locations
Non-hydrostatic modelling

Validations

Long wave propagation in the swash zone

Nha Trang project (Vietnam/France): high frequency and high resolution swash database

→ see Almar et al. (IAHR-APD 2014)
Conclusion and perspectives

- non-hydrostatic and dispersive effects play a significant role in long wave dynamics in coastal and estuarine environments

- need to reassess tidal bore occurrence in meso and macro-tidal estuaries worldwide (including Asian & Pacific estuaries)
  → impact on estuarine ecosystems
Conclusion and perspectives

- A new approach for long wave modelling

\[ \partial_t h + \nabla \cdot (hu) = 0 \]

\[ \partial_t (hu) + \nabla \cdot (hu \otimes u) + \nabla \left( \frac{1}{2} gh^2 \right) = -gh \nabla b + \frac{1}{\alpha} gh \nabla \zeta - \left( I + \alpha h \mathcal{T} \frac{1}{h} \right)^{-1} \left[ \frac{1}{\alpha} gh \nabla \zeta + h \mathcal{Q}_1(u) \right] \]

- new mathematical formulation
  → easy to implement in existing NSWE models

- hybrid approach SGN/NSWE
  → wave transformation and wave breaking

- Development of Finite Element methods on unstructured grid

Mario Ricchiuto (INRIA, Bordeaux)
Thank you for your attention