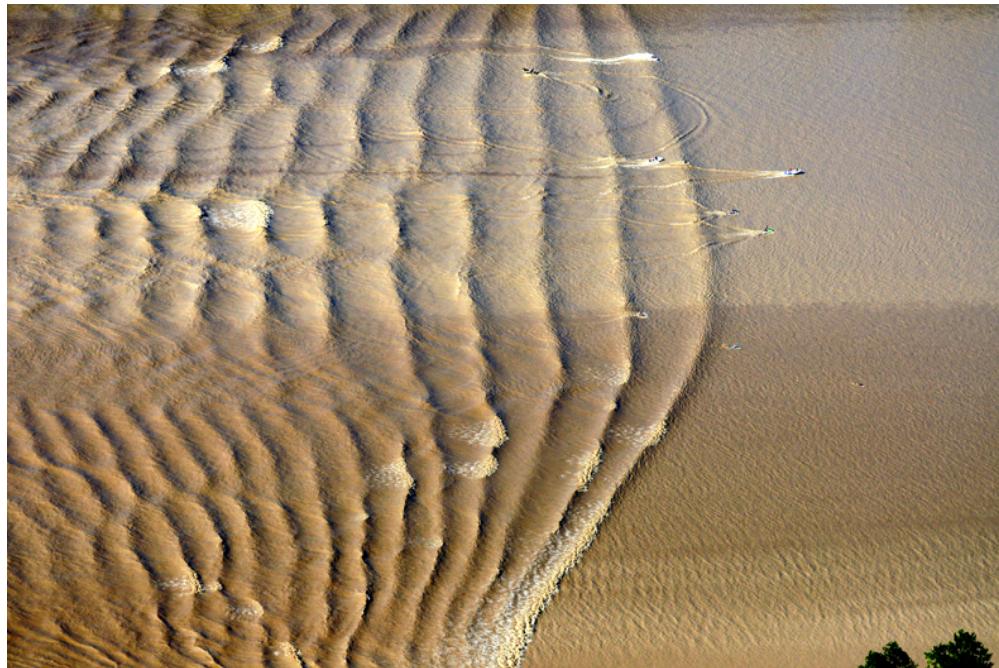


Tsunami-like bore formation in coastal and estuarine environments

Philippe Bonneton

EPOC/METHYS, CNRS, Bordeaux Univ.

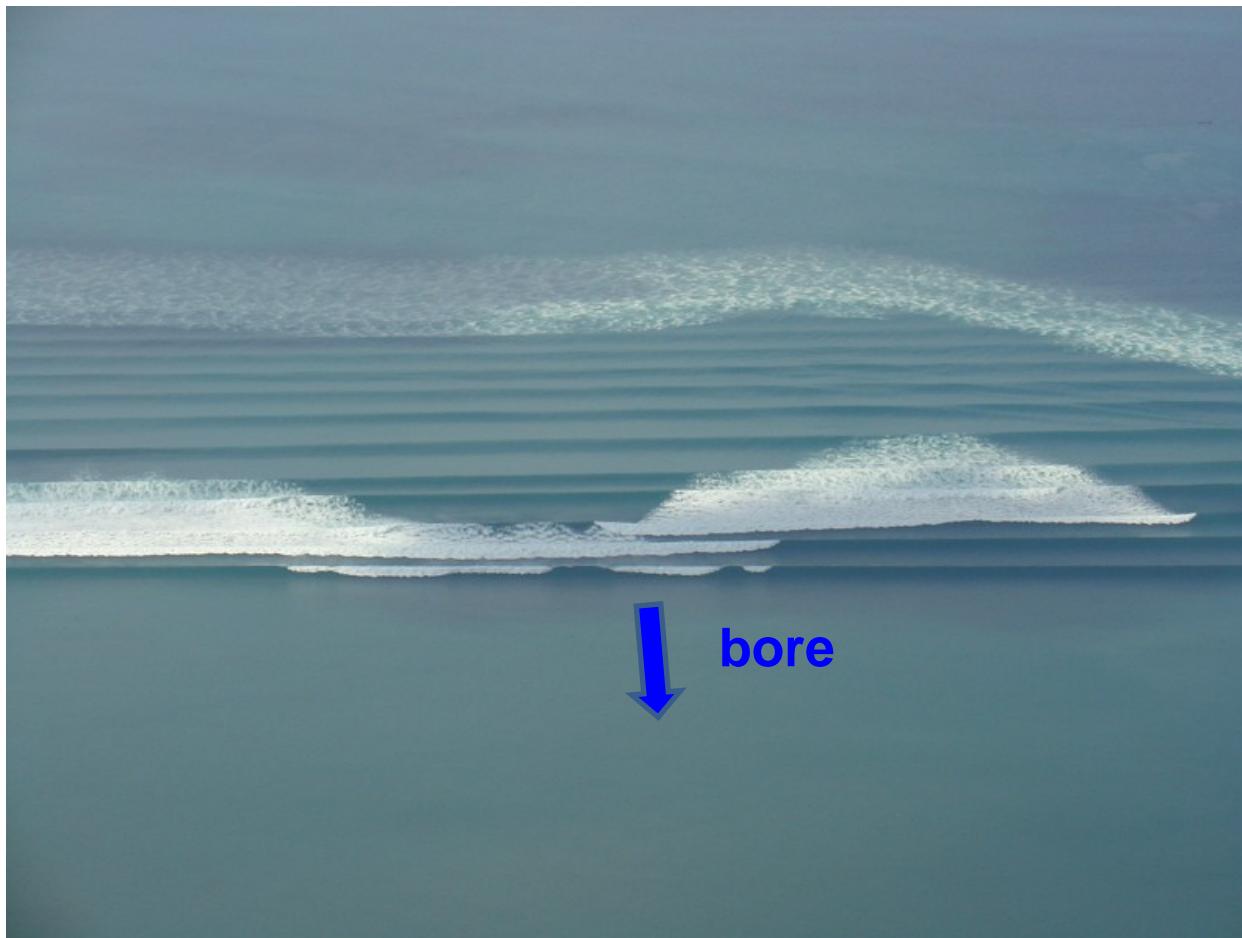


Tidal bore – Gironde – Lavigne Y.

Introduction

Long waves

Tsunamis, tides, infragravity waves, meteorological tsunamis, ...

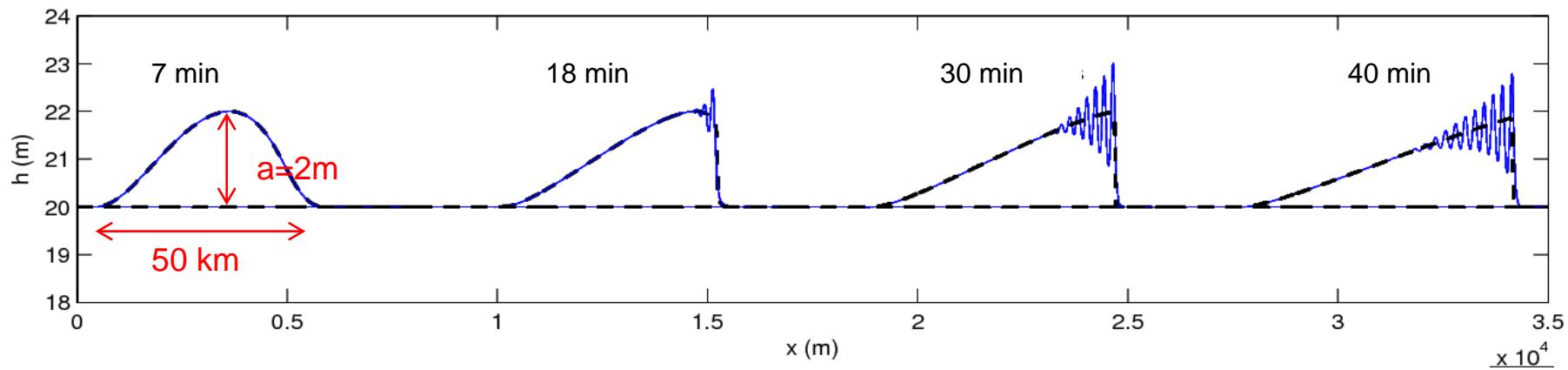


Sumatra 2004 tsunami reaching the coast of Thailand

Introduction

Long waves

Tsunamis, tides, infragravity waves, meteorological tsunamis, ...



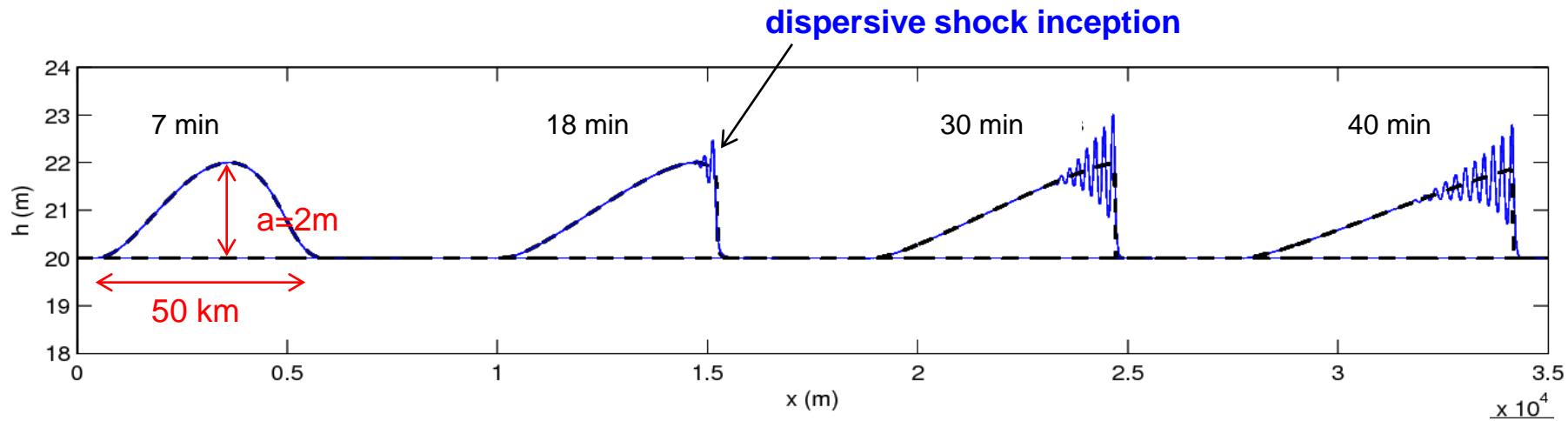
Serre Green Naghdi model

Tissier , Bonneton et al., JCR2011

Introduction

Long waves

Tsunamis, tides, infragravity waves, meteorological tsunamis, ...



⇒ bore formation

Introduction

Tsunami bore



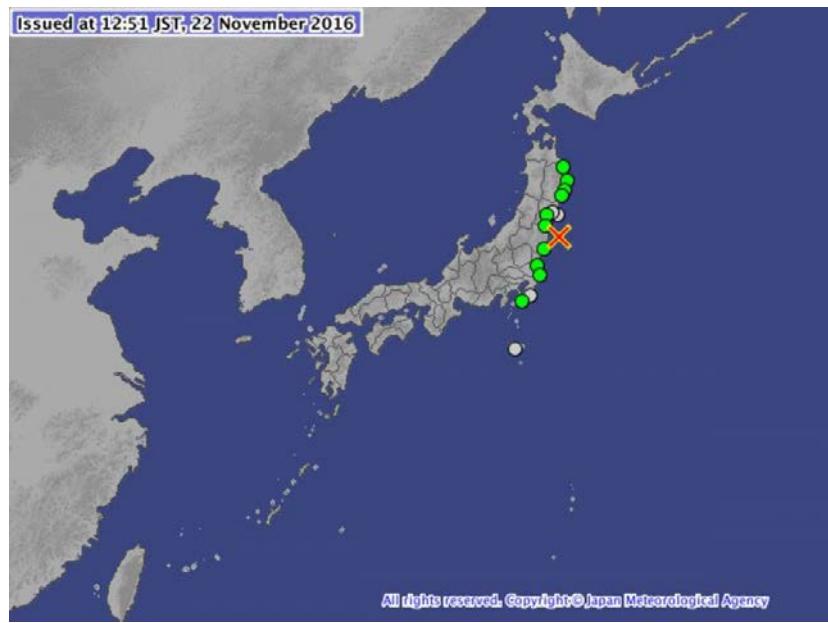
Sumatra 2004 tsunami reaching
the coast of Thailand

references:

- *Grue et al. 2008*
- *Madsen et al. 2008*

Introduction

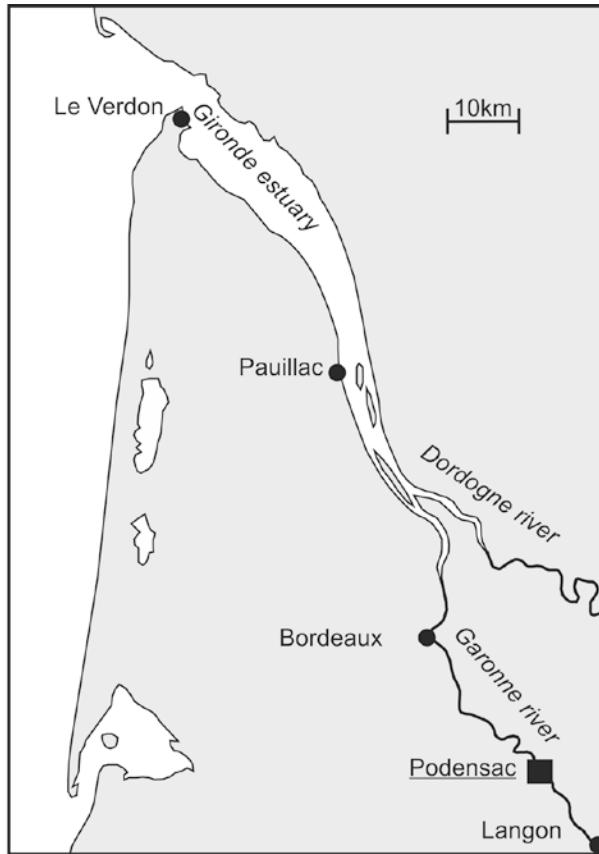
Tsunami bore



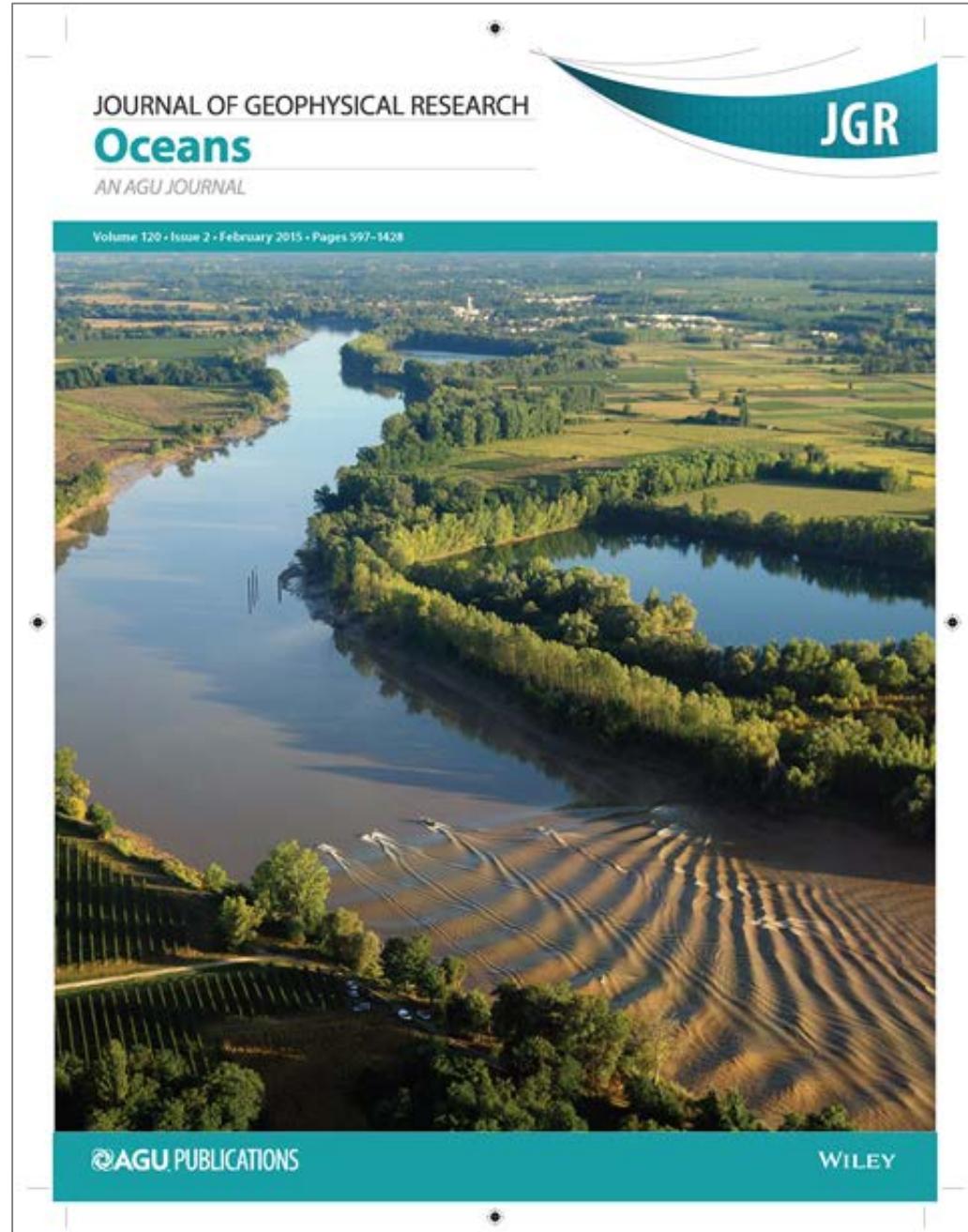
21st November 2016, Sunaoshi River in Tagajo city, Japan (earthquake 7.4)

Introduction

Tidal bore



Bonneton et al. JGR 2015



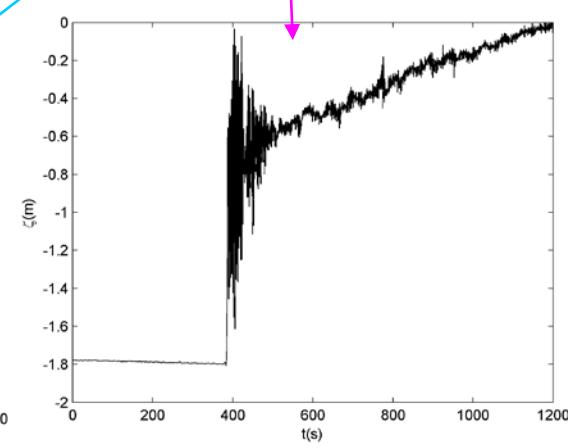
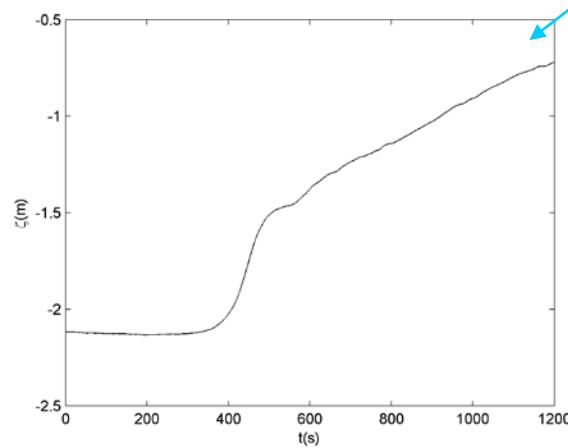
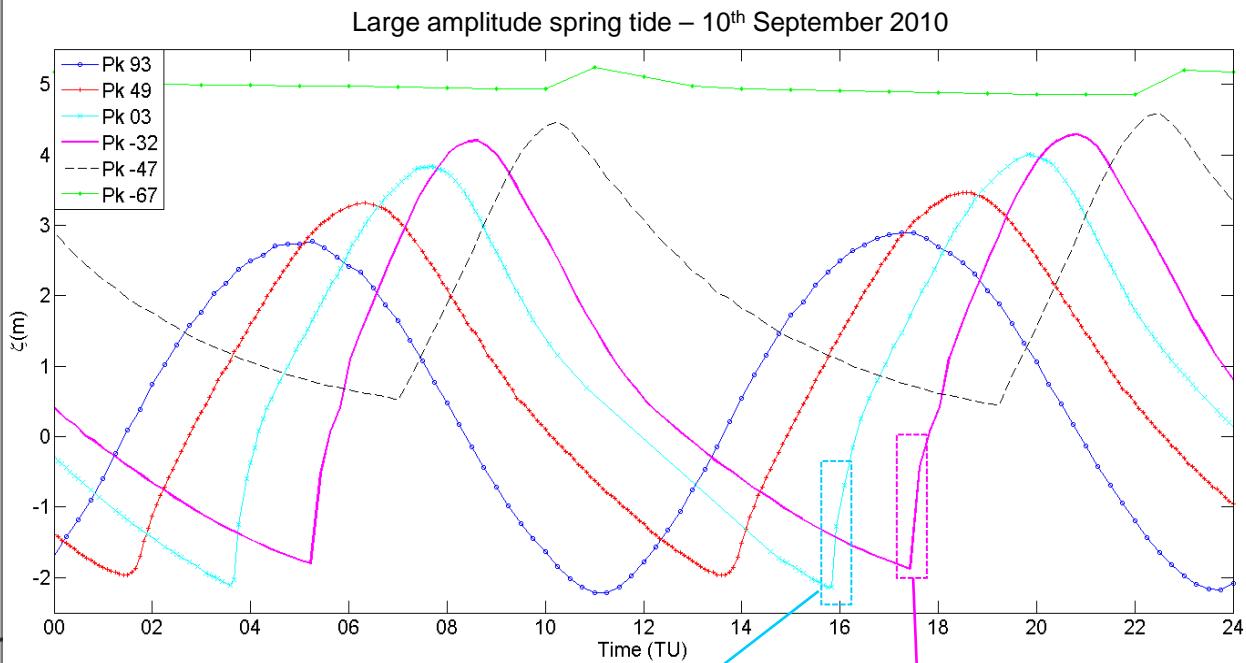
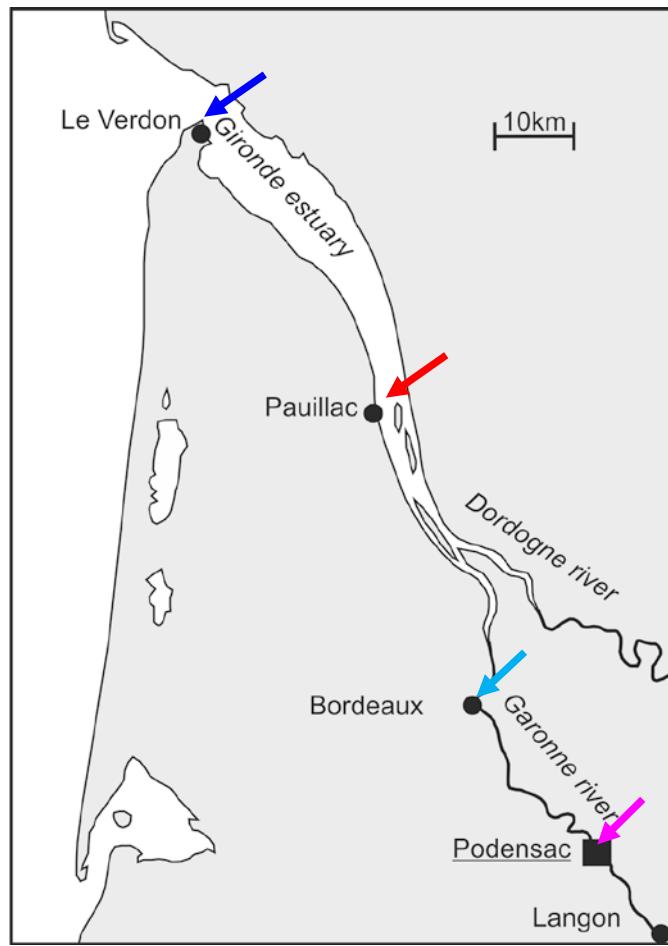


Gironde estuary, Saint Pardon, Dordogne

<https://vimeo.com/106090912>, Jean-Marc Chauvet, Septembre 2014

Introduction

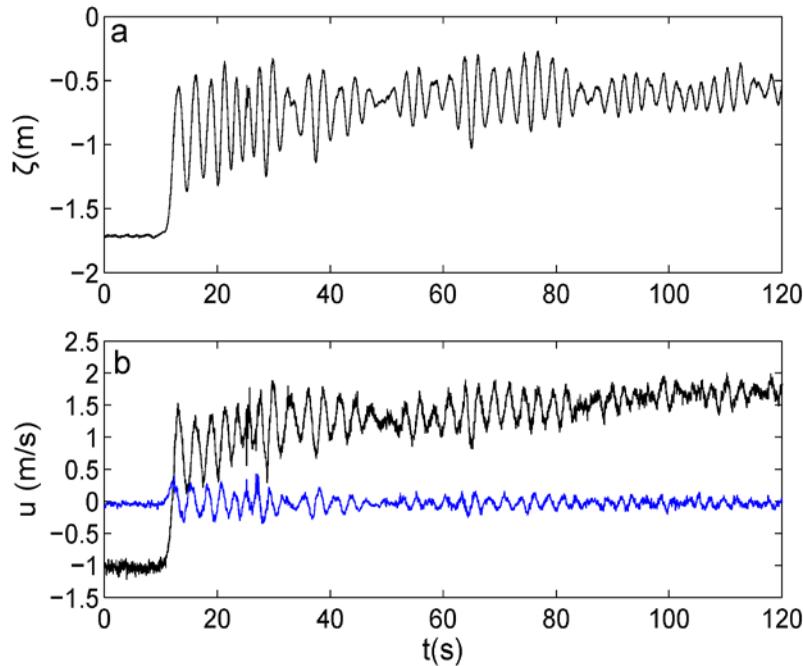
Tidal bore



⇒ Strong impact on coastal and estuarine environments

Introduction

□ Sediment transport and erosion



Impact of tsunami-like bores

Tidal Bores, Bonneton et al. 2015



□ Impact on marine structures and buildings

Type 1 Overflow



Type 2 Bore



Tsunamis, Arikawa et al. 2013



Conditions for tsunami-like bore formation in coastal and estuarine environments



tsunami bore



tidal bore

□ field measurements

- Bonneton N., Castelle B., Detandt G., Parisot J-P., Sottolichio A.
(EPOC, METHYS team, Bordeaux)
- Frappart F., Roussel N., Darrozes J. (OMP, Toulouse)
- Martins K. (Bath University)

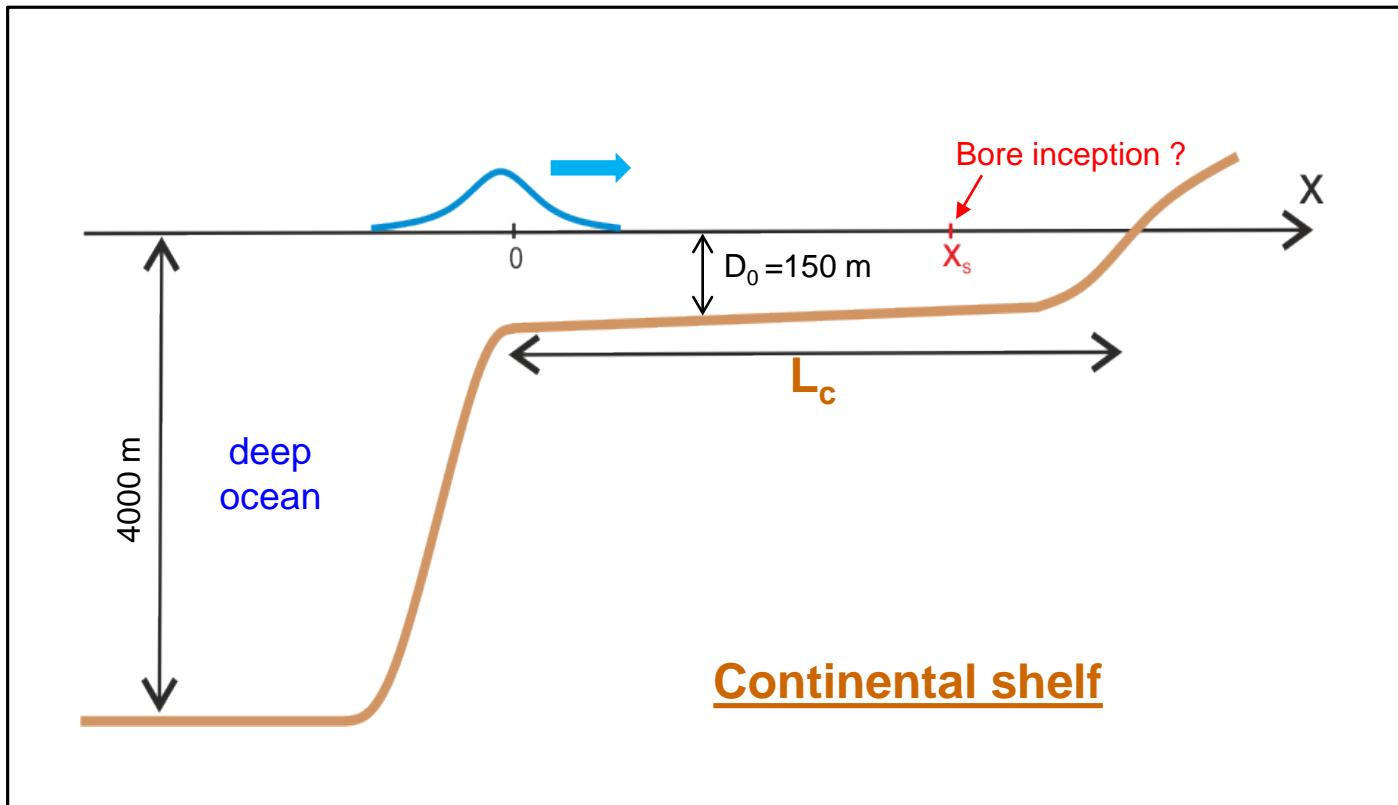
□ long wave modeling

- Ricchuito M., Arpaia L., Filippini A. (INRIA, Bordeaux)
- Lannes D. (IMB, Bordeaux)
- Marche F. (IMAG, Montpellier),
- Tissier M. (TU Delft, Netherlands)

- ❑ **Introduction**
- ❑ **Basic conditions for bore formation
in the coastal zone**
- ❑ **Conditions for tidal bore formation
in estuaries**
- ❑ **Conclusion and perspectives**

- ❑ Introduction
- ❑ Basic conditions for bore formation
in the coastal zone
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in estuaries
- ❑ Conclusion and perspectives

Basic conditions for bore formation



- the continental shelf is relatively flat

- $D_0, A_0, \omega_0 = 2\pi/T_0$

$$L_{w0} = \frac{\sqrt{gD_0}}{\omega_0}$$

$$\epsilon_0 = \frac{A_0}{D_0} \quad \mu_0 = \frac{D_0^2}{L_{w0}^2}$$

Basic conditions for bore formation

$$\mu_0 \ll 1 \quad \varepsilon_0 = O(1)$$

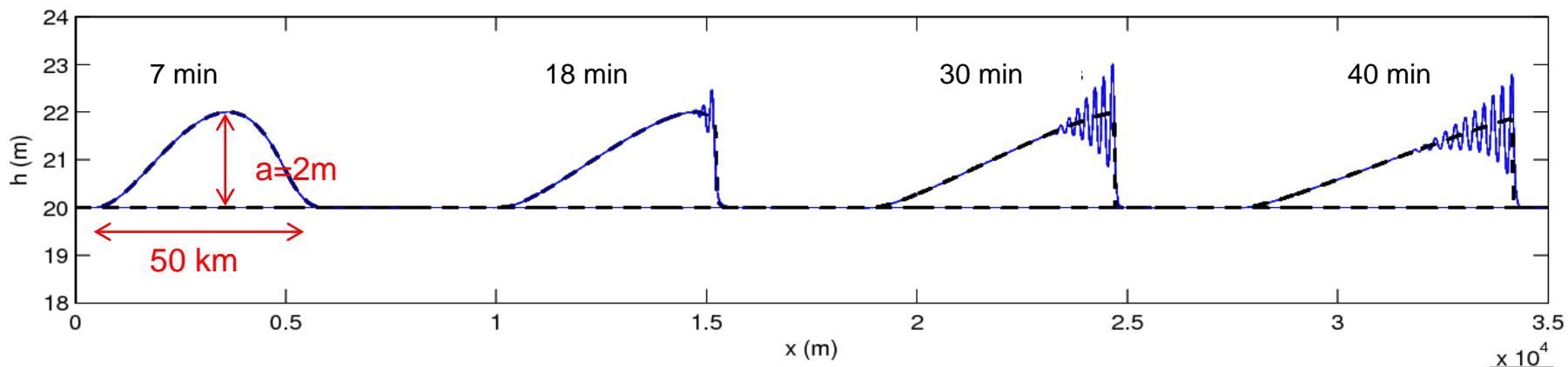
$$\begin{aligned}\frac{\partial \zeta}{\partial t} + \vec{\nabla} \cdot ((1 + \epsilon_0 \zeta) \vec{u}) &= 0 \\ \frac{\partial \vec{u}}{\partial t} + \epsilon_0 (\vec{u} \cdot \vec{\nabla}) \vec{u} + \vec{\nabla} \zeta &= \mu_0 D\end{aligned}$$

Serre / Green Naghdi equations

Basic conditions for bore formation

$$\mu_0 \ll 1 \quad \epsilon_0 = O(1)$$

$$\frac{\partial \zeta}{\partial t} + \vec{\nabla} \cdot ((1 + \epsilon_0 \zeta) \vec{u}) = 0$$
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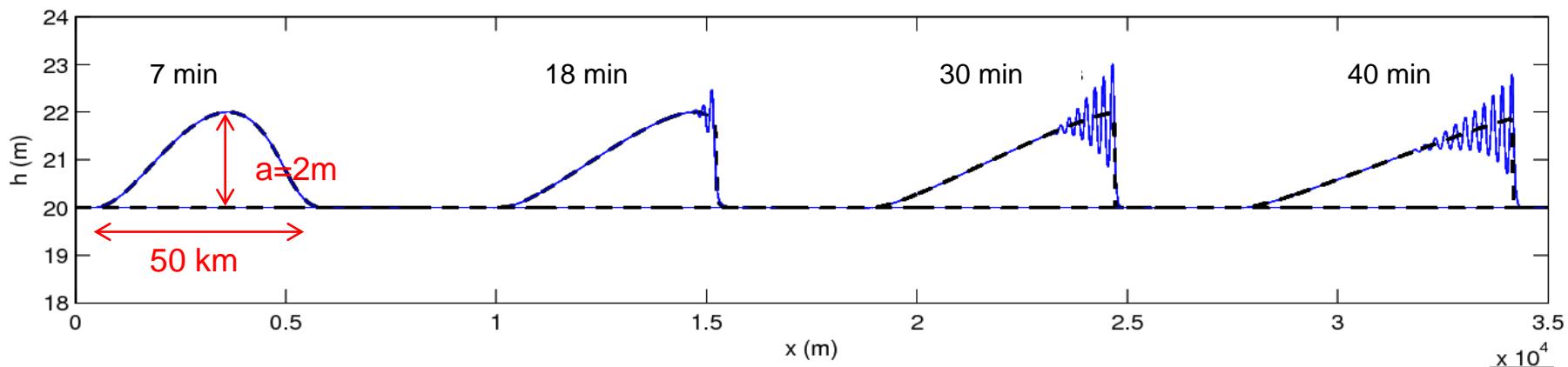
Serre Green Naghdi model

Tissier , Bonneton et al., JCR2011

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Serre Green Naghdi model

Tissier , Bonneton et al., JCR2011

Basic conditions for bore formation

$$\begin{aligned}\frac{\partial}{\partial t}(u - 2c) + (u - c)\frac{\partial}{\partial x}(u - 2c) &= 0 \\ \frac{\partial}{\partial t}(u + 2c) + (u + c)\frac{\partial}{\partial x}(u + 2c) &= 0\end{aligned}$$

$$c = (gh)^{1/2} \quad c_0 = (gD_0)^{1/2}$$

One way $\Rightarrow u - 2c = -2c_0$

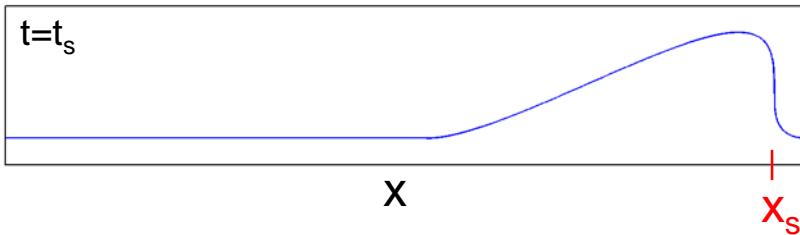
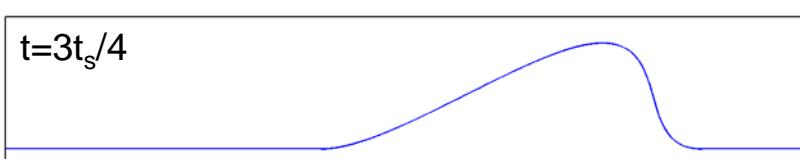
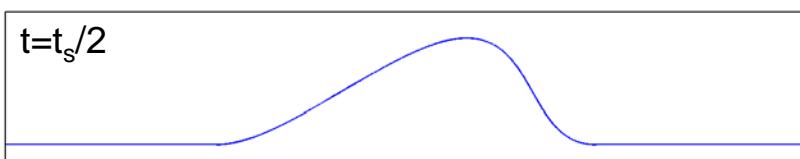
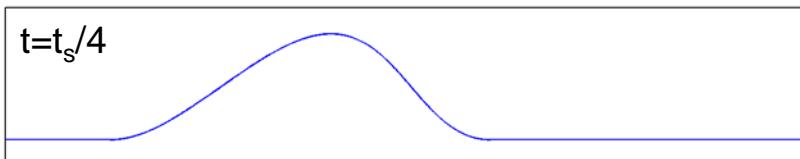
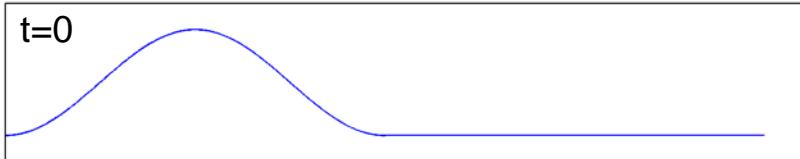
$$\frac{\partial h}{\partial t} + C_h(h)\frac{\partial h}{\partial x} = 0 \quad C_h = 3c - 2c_0$$

$$\left\{ \begin{array}{l} h(x, t) = h(x_0, t = 0) \quad \text{along} \\ \frac{dx}{dt} = C_h \quad \text{or} \quad x = x_0 + C_h(x_0, t = 0)t \end{array} \right.$$

$$h(x, t) = h_0(x - C_h(x_0)t)$$

Basic conditions for bore formation

$$h(x, t) = h_0(x - C_h(x_0)t)$$

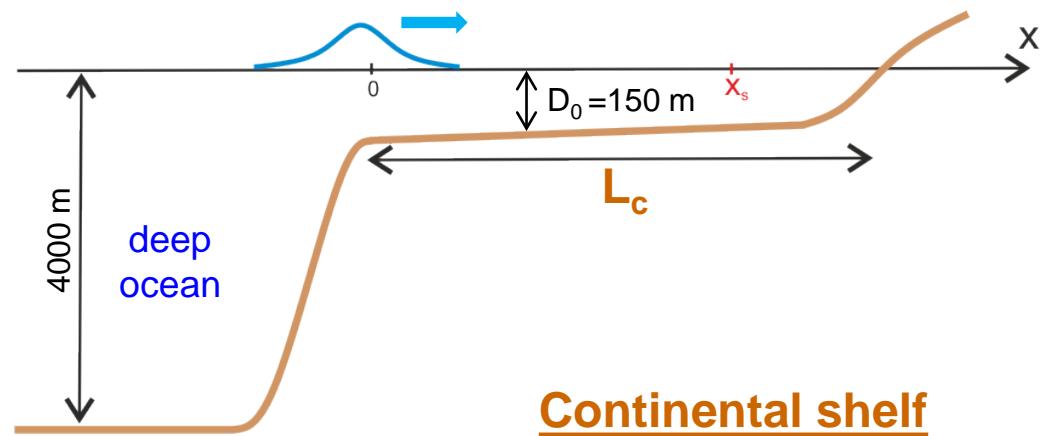


$$\frac{\partial h}{\partial x} = \frac{\frac{\partial h_0}{\partial x_0}}{1 + \frac{\partial C_h}{\partial x_0} t} = \frac{\frac{\partial h_0}{\partial x_0}}{1 + \frac{3c_0}{2\sqrt{D_0 h_0}} \frac{\partial h_0}{\partial x_0} t}$$

$$t_s = -\frac{2}{3} \frac{D_0}{c_0 \frac{\partial h_0}{\partial x_0}} \quad \Rightarrow \quad x_s = -\frac{2}{3} \frac{D_0 C_{h_s}}{c_0 \frac{\partial h_0}{\partial x_0}}$$

$$x_s \simeq -\frac{2}{3} \frac{D_0}{\frac{\partial h_0}{\partial x_0}}_s$$

Basic conditions for bore formation

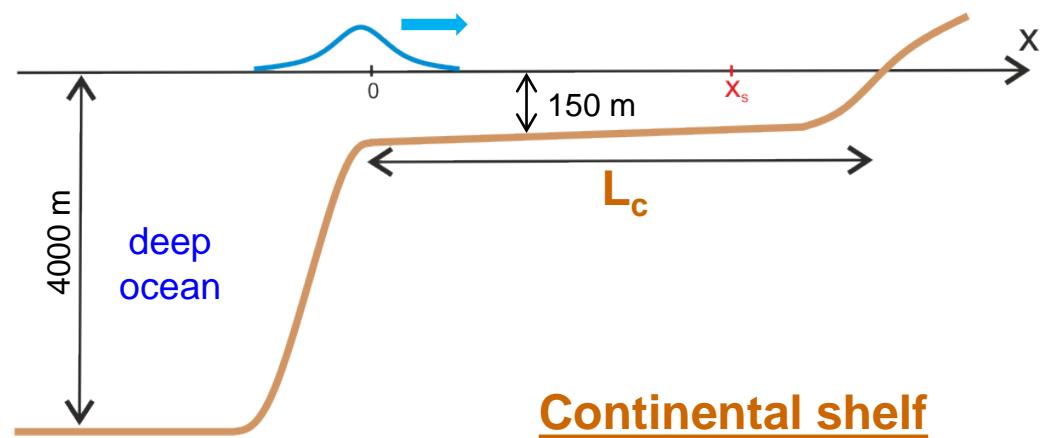


$$h(x_0, t = 0) = D_0 + A_0(1 + \sin \frac{x_0}{L_{w0}})$$

$$x_s = \frac{2 L_{w0}}{3 \epsilon_0}$$

see Madsen et al 2008

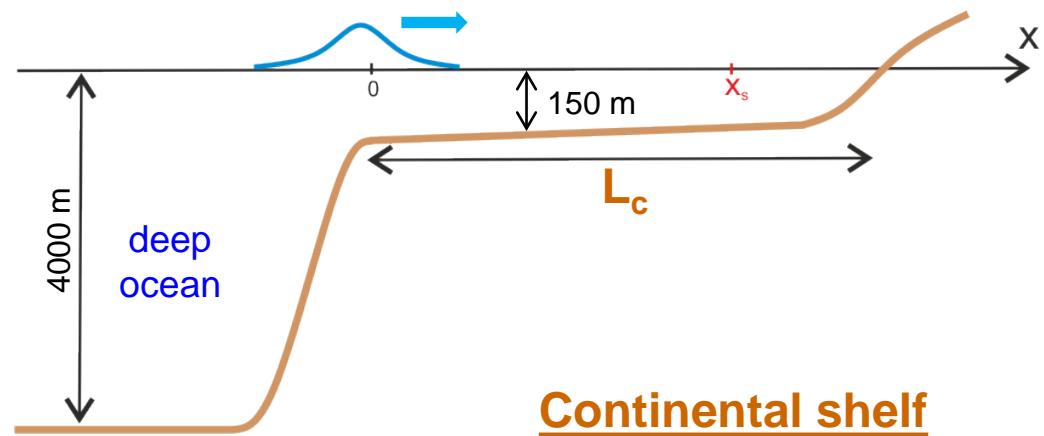
Basic conditions for bore formation



$$h(x_0, t = 0) = D_0 + A_0(1 + \sin \frac{x_0}{L_{w0}})$$

$$x_s = \frac{2 L_{w0}}{3 \epsilon_0} < L_c$$

Basic conditions for bore formation

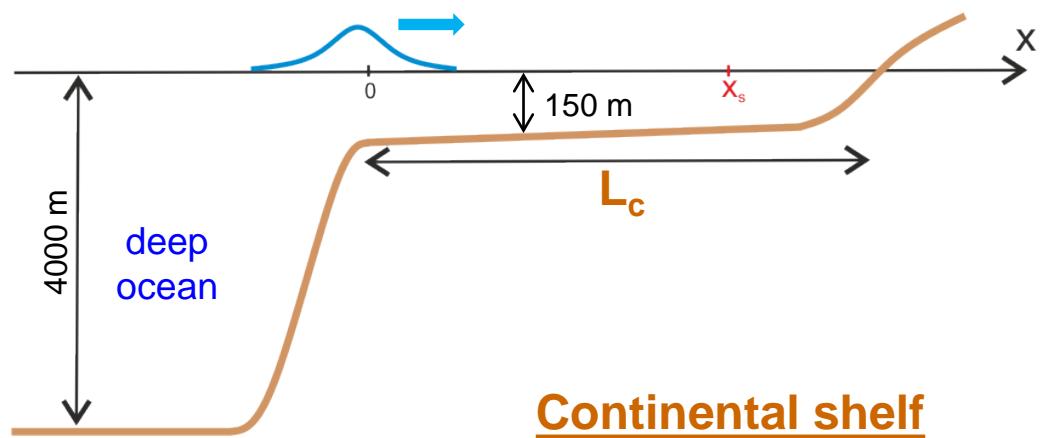


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Basic conditions for bore formation



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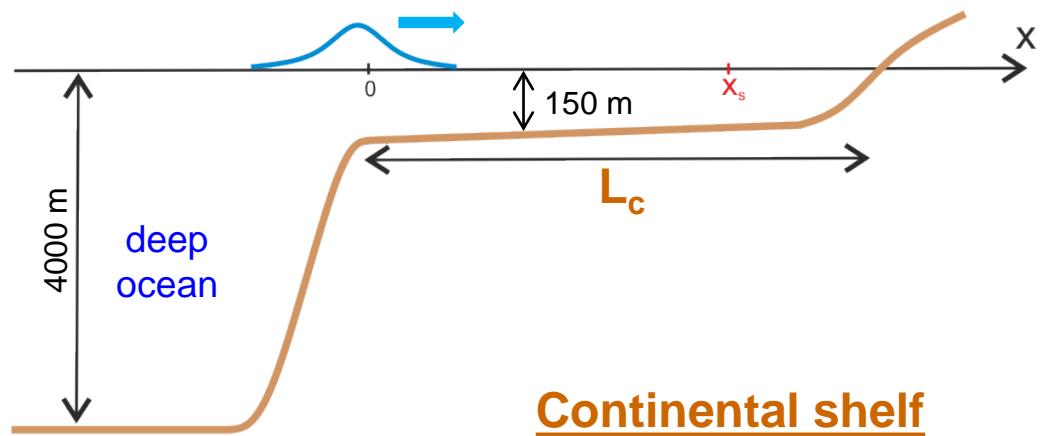
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$$\delta_0 = \frac{L_{w0}}{L_c} < \frac{3}{2} \epsilon_0$$

Continental shelves $D_0 \approx 150$ m

- tsunamis: $A_0 \approx 2$ m, $T_0 \approx 25$ min $\Rightarrow x_s = 460$ km
- tides: $T_0 \approx 744$ min $\Rightarrow x_s \gg L_c \rightarrow$ no tidal bore in open continental shelf water

Basic conditions for bore formation



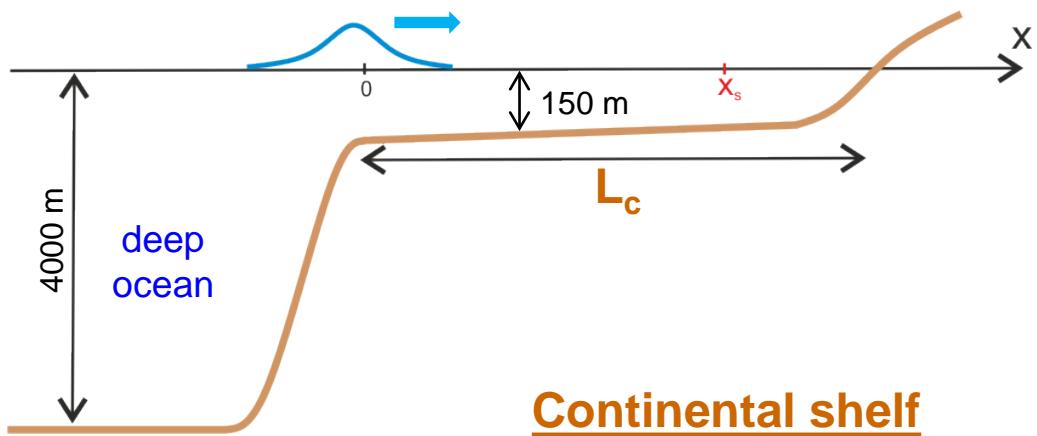
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- tsunamis: bores may occur in large and shallow (few tens of m) coastal environments:
marine coastal plains (e.g.: deltas, alluvial estuaries) or
carbonate platforms (e.g.: coral reef systems)

Basic conditions for bore formation

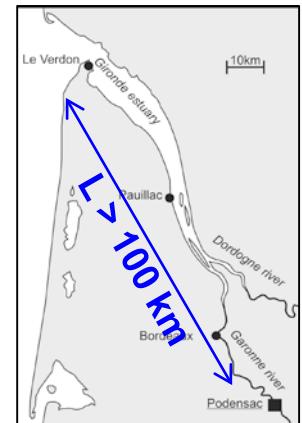


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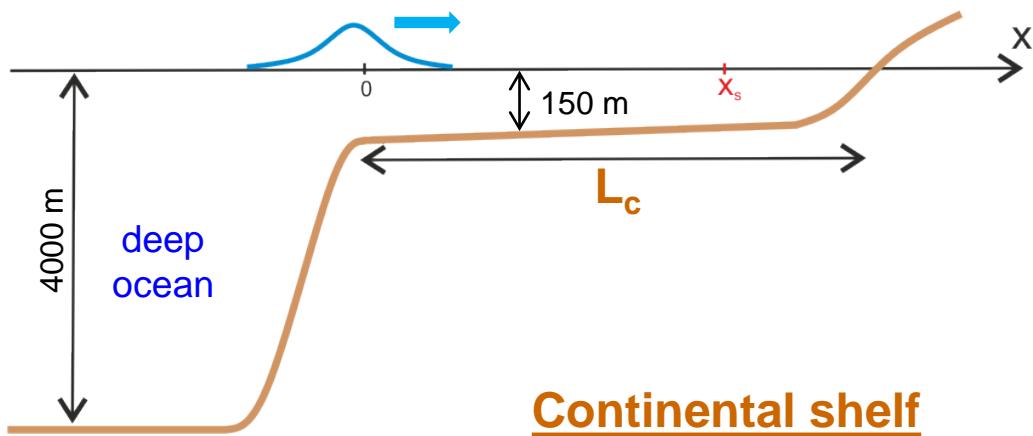
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marine coastal plains (e.g.: deltas, alluvial estuaries) or
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- tides: bores can occur in **long shallow alluvial estuaries**
 $L \approx 100 \text{ km}$ $D_0 \approx 10 \text{ m}$



Basic conditions for bore formation



$$h(x_0, t = 0) = D_0 + A_0(1 + \sin \frac{x_0}{L_{w0}})$$

$$x_s = \frac{2L_{w0}}{3\epsilon_0} < L_c$$

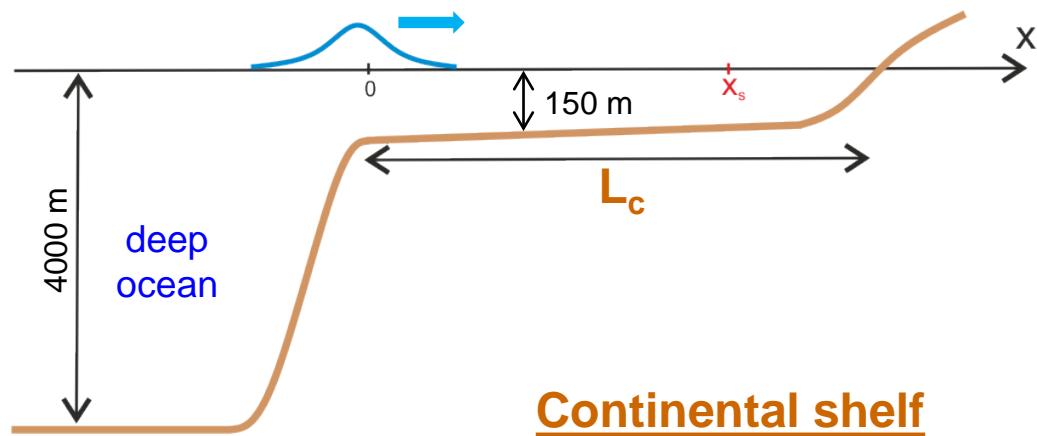
$$\delta_0 = \frac{L_{w0}}{L_c} < \frac{3}{2}\epsilon_0$$

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marine coastal plains (e.g.: deltas, alluvial estuaries) or
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- tides: bores can occur in **long shallow alluvial estuaries**

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} + C_{f0} \frac{|u|u}{D} = 0$$

$$\mathcal{D}_i \sim \frac{C_{f0} \frac{|u|u}{D}}{\frac{\partial u}{\partial t}} \sim C_{f0} \epsilon_0 \frac{L_{w0}}{D_0}$$

Basic conditions for bore formation



$$h(x_0, t = 0) = D_0 + A_0(1 + \sin \frac{x_0}{L_{w0}})$$

$$x_s = \frac{2 L_{w0}}{3 \epsilon_0} < L_c$$

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marine coastal plains (e.g.: deltas, alluvial estuaries) or
carbonate platforms (e.g.: coral reef systems)
- tides: bores can occur in **long shallow alluvial estuaries**

in such shallow environments friction plays a significant role

Outline

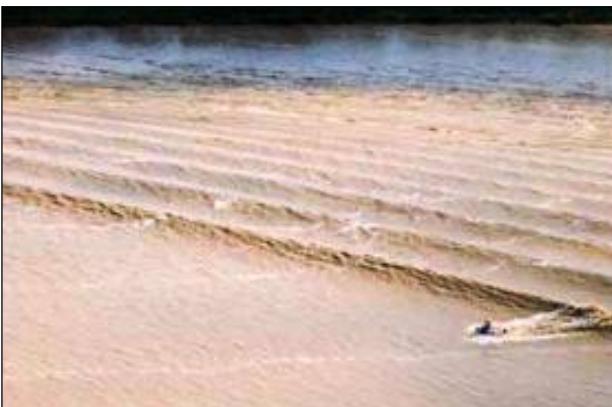
- ❑ Introduction
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in estuaries
- ❑ Conclusion and perspectives

Conditions for tidal bore formation

Worldwide tidal bores



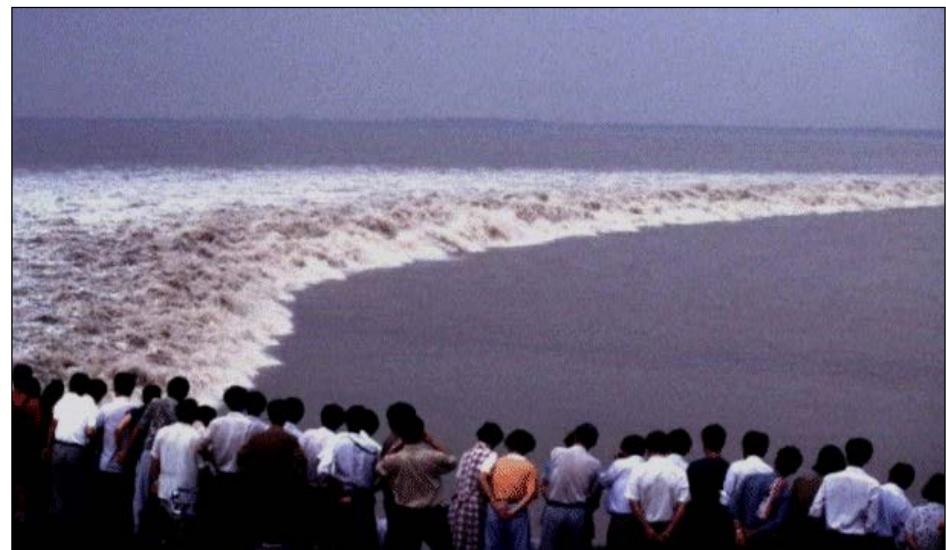
Severn River - England



Amazon River – Brazil (Pororoca)



Kampar River – Sumatra (Bono)



Qiantang River – China

©Lawrence

Conditions for tidal bore formation

Worldwide tidal bores



Gironde/Garonne/Dordogne estuary – France

3 field campaigns : a unique long-term high-frequency database



Bonneton et al. JGR 2015

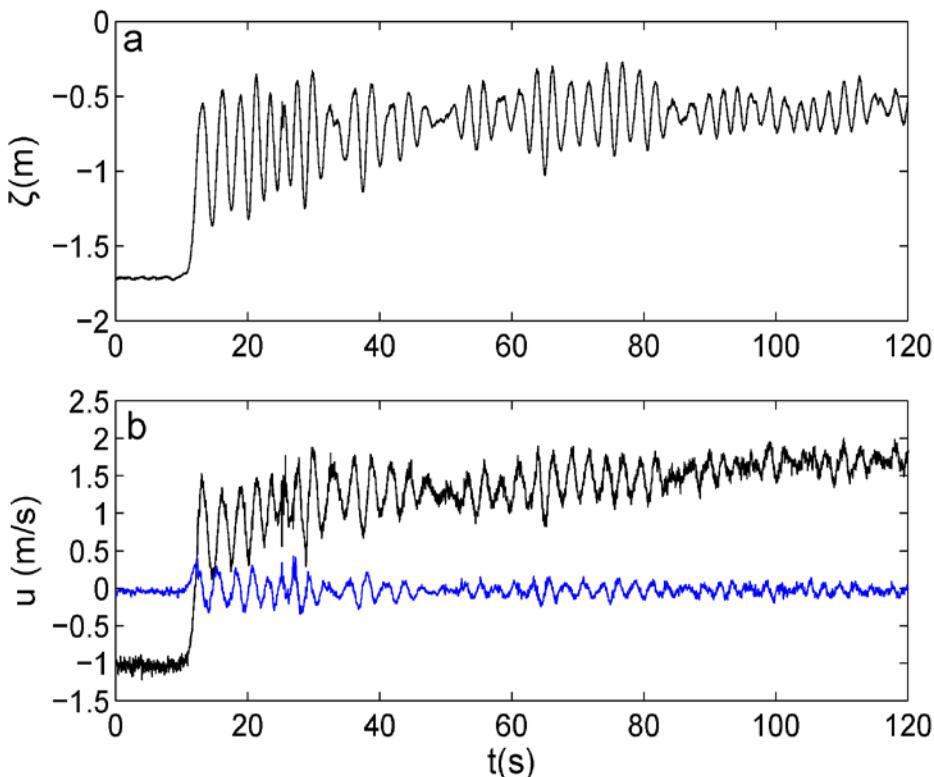
Conditions for tidal bore formation

Worldwide tidal bores



Gironde/Garonne/Dordogne estuary – France

3 field campaigns : a unique long-term high-frequency database



Bonneton et al. JGR 2015

- Large tidal range ($Tr_0=2A_0$) → Chanson (2012) : $Tr_0 > 4.5-6 \text{ m}$

- Small water depth

- Large-scale funnel-shaped estuaries

} coastal plain alluvial estuaries

⇒ Scaling analysis

- Tide propagation in convergent estuaries :

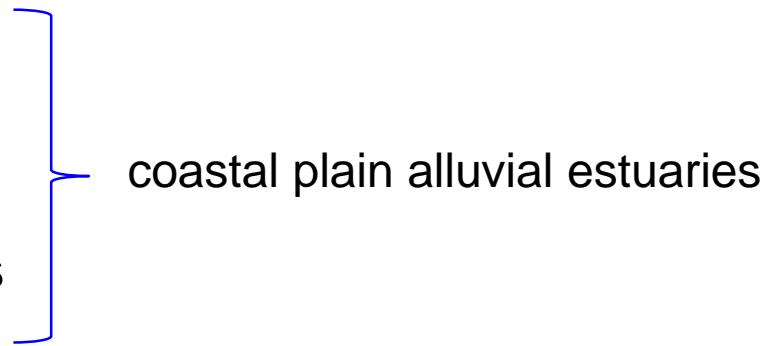
Parker 1991, Lanzoni and Seminara 1998, Toffolon et al. 2006

- First scaling analysis of the tidal bore problem

Large tidal range

Small water depth

Large-scale funnel-shaped estuaries



coastal plain alluvial estuaries

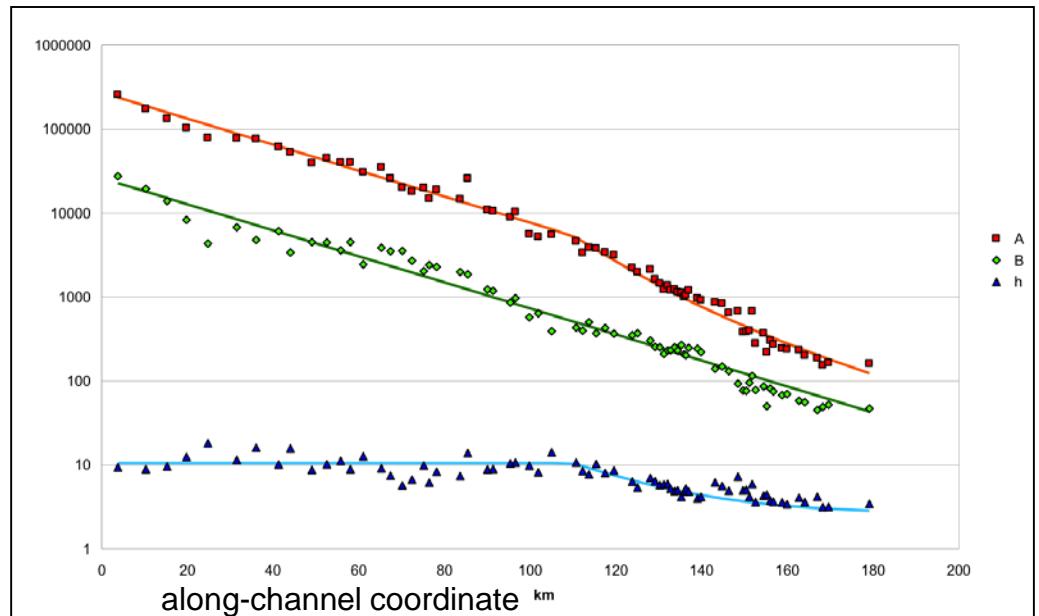
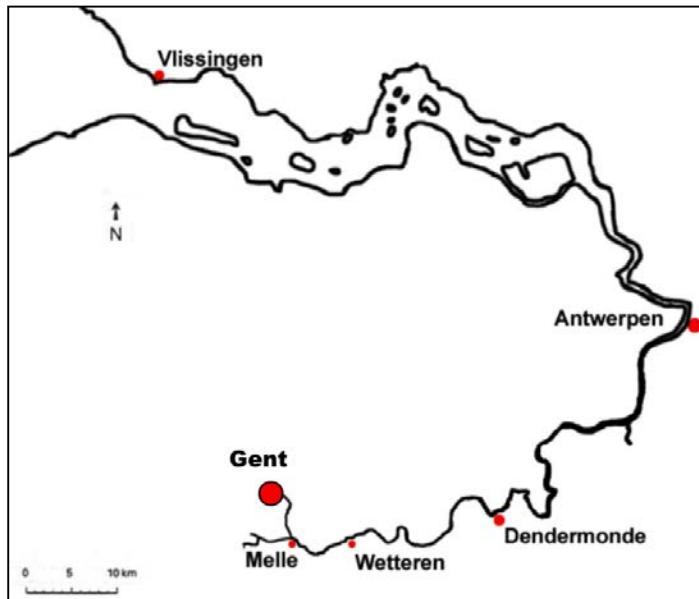
⇒ Scaling analysis

Identify the characteristic scales of the problem:

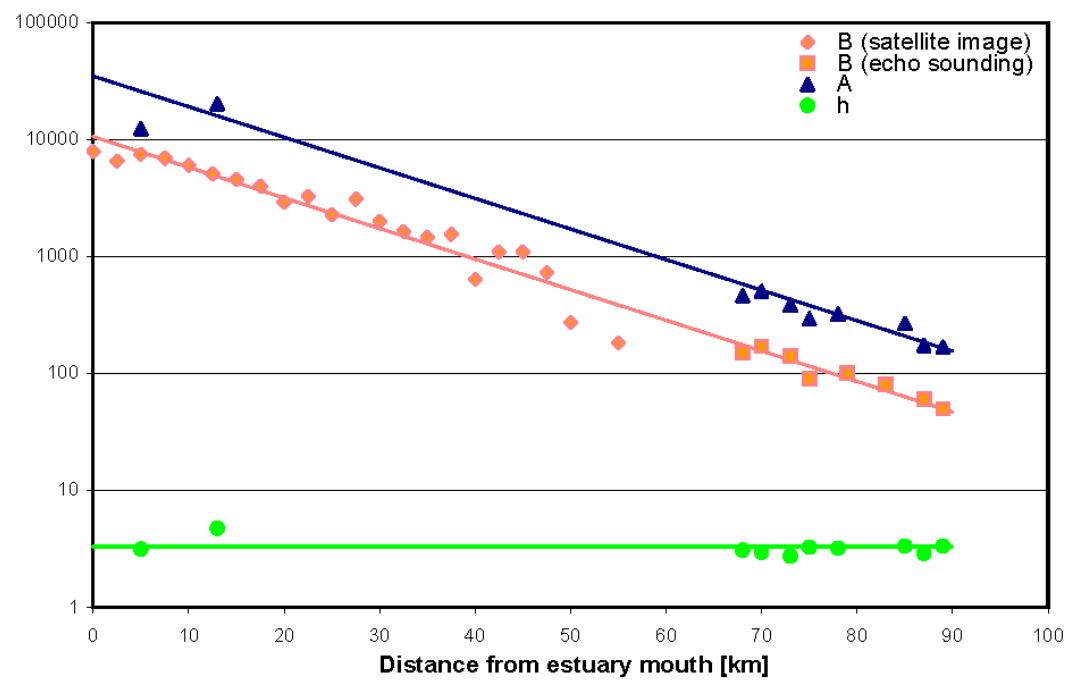
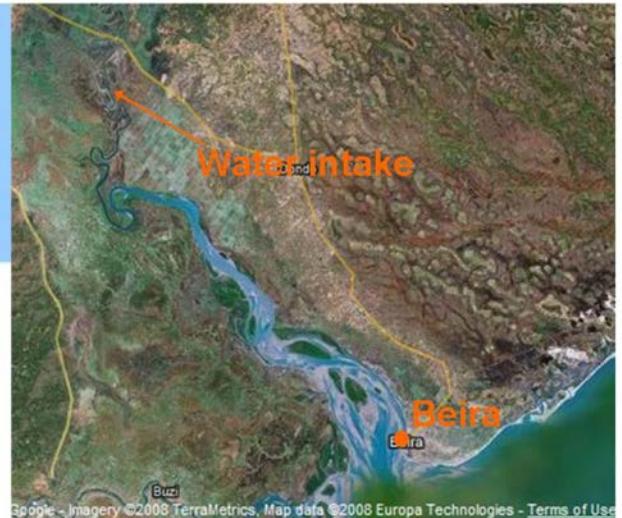
- morphology of alluvial estuaries
- tidal wave forcing

Tide-dominated alluvial estuaries show
many morphological similarities all over the world

Scheldt estuary



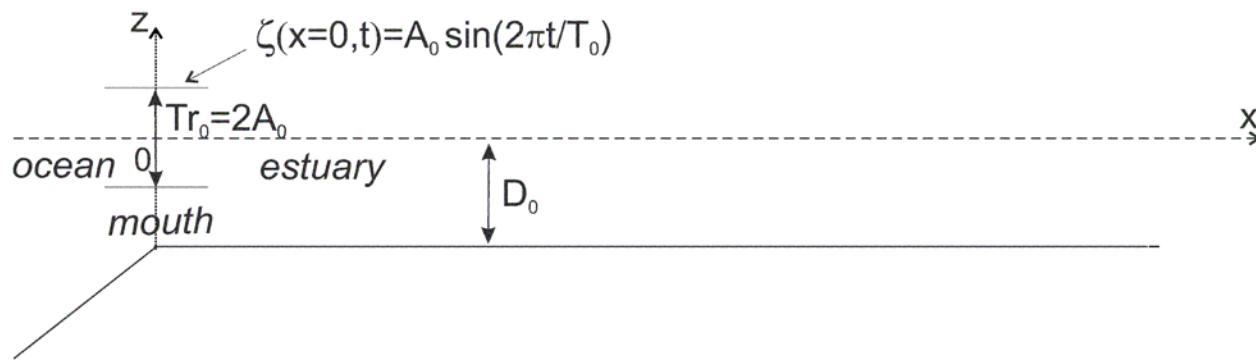
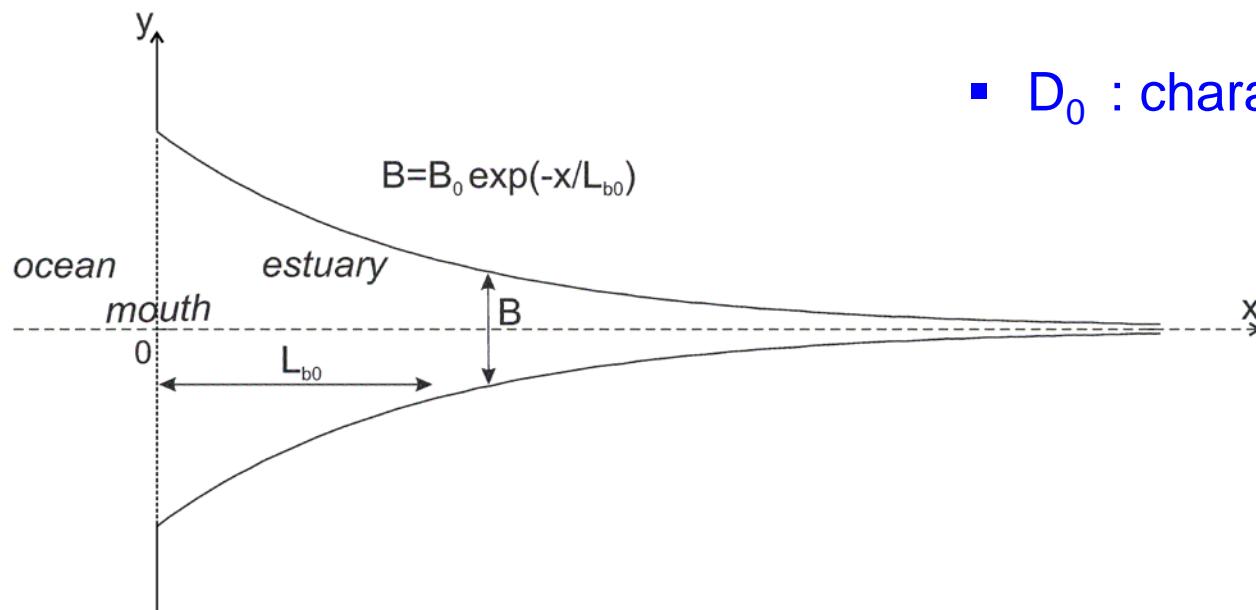
Pungue estuary



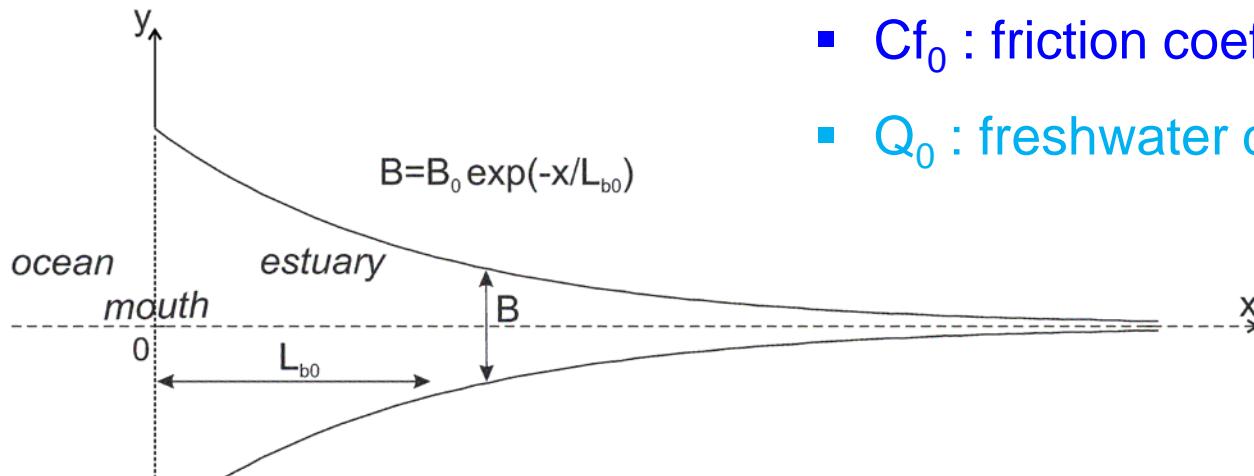
Graas et al. 2008

$$B = B_0 e^{-x/L_{b0}}$$

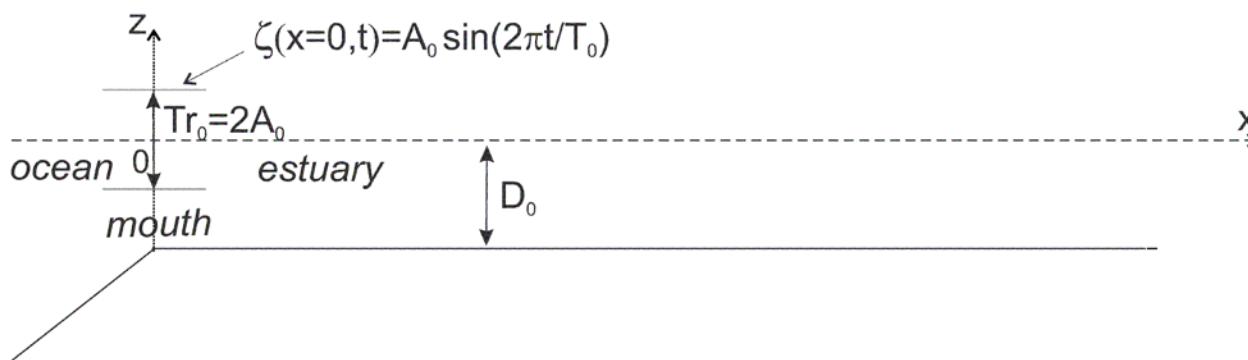
- L_{b0} : convergence length
- D_0 : characteristic water depth



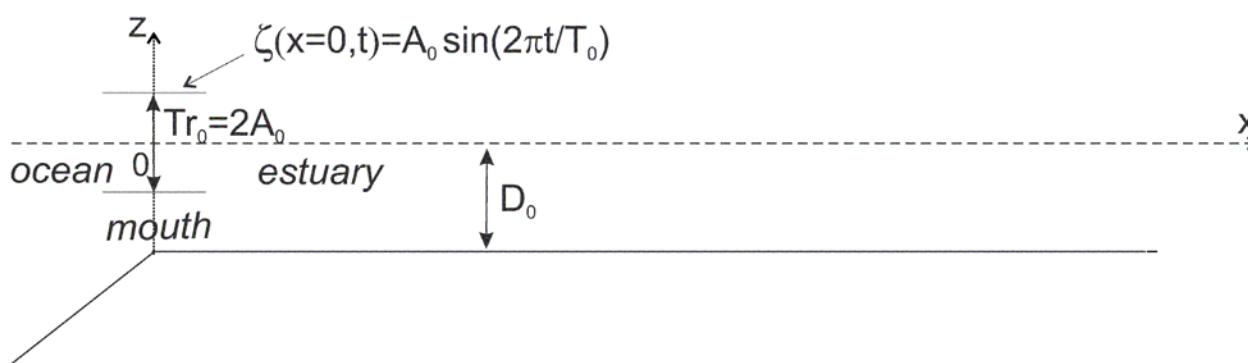
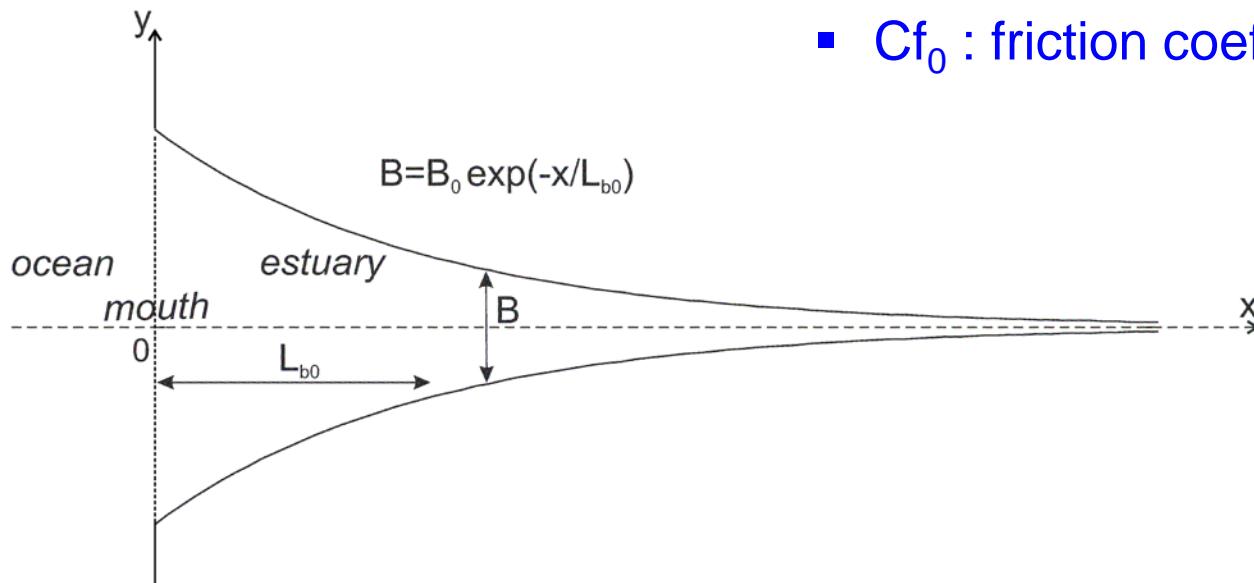
- $A_0 = T r_0 / 2$ (mean spring tidal amplitude)
- $T_0 = 12.4 \text{ h} \rightarrow L_{w0} = \sqrt{g D_0} / \omega_0$



- $C f_0$: friction coefficient
- Q_0 : freshwater discharge

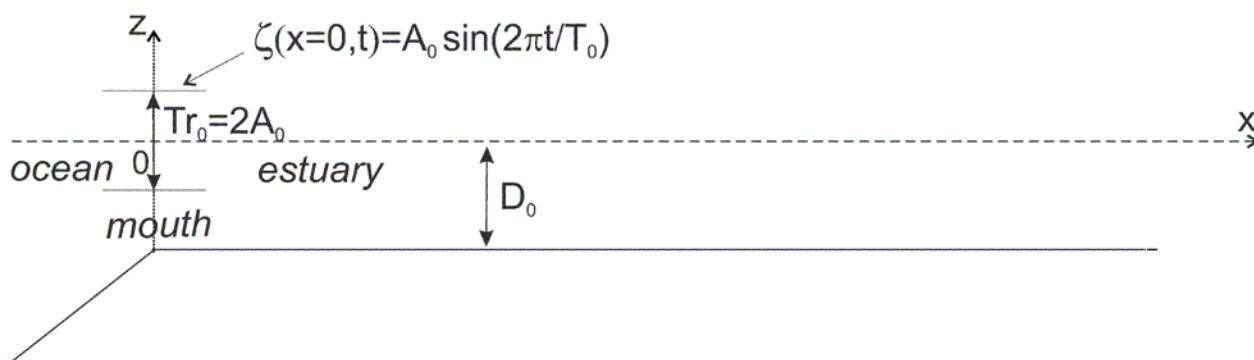
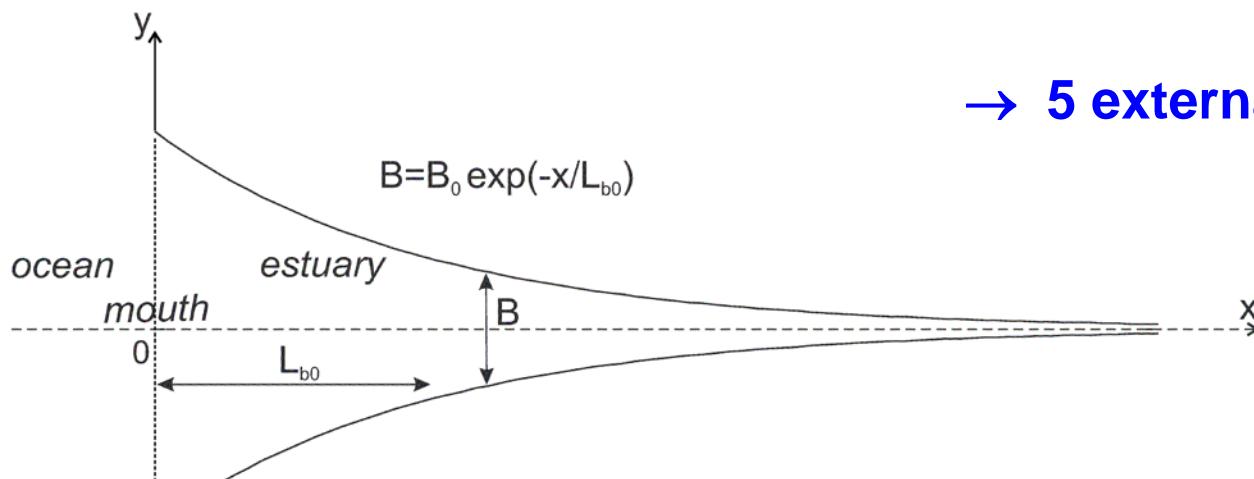


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- $T_0 = 12.4 \text{ h} \rightarrow L_{w0} = \sqrt{g D_0} / \omega_0$
- $C f_0$: friction coefficient



- L_{b0}
- D_0
- A_0
- L_{w0}
- Cf_0

→ 5 external variables



$$\begin{aligned}\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + D \frac{\partial u}{\partial x} + \frac{u}{B} \frac{\partial \mathcal{A}}{\partial x} \Big|_{z=\zeta} &= 0 \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} + C_{f0} \frac{|u|u}{D} &= 0\end{aligned}$$

$$B = B_0 \exp\left(-\frac{x}{L_{b0}}\right) \quad \frac{1}{B} \frac{\partial \mathcal{A}}{\partial x} \Big|_{z=\zeta} = -\frac{D}{L_{b0}}$$

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$$B = B_0 \exp\left(-\frac{x}{L_{b0}}\right) \quad \frac{1}{B} \frac{\partial \mathcal{A}}{\partial x} \Big|_{z=\zeta} = -\frac{D}{L_{b0}}$$

$$t' = \frac{t}{T_0/2\pi}, \quad D' = \frac{D}{D_0}, \quad \zeta' = \frac{\zeta}{A_0}, \quad x' = \frac{x}{L_0}, \quad u' = \frac{u}{U_0}.$$

$$\frac{\partial \zeta}{\partial t} + \mu \lambda \left(\epsilon_0 u \frac{\partial \zeta}{\partial x} + D \frac{\partial u}{\partial x} \right) - \mu \delta_0 u D = 0$$

$$\frac{\partial u}{\partial t} + \mu \lambda \epsilon_0 u \frac{\partial u}{\partial x} + \frac{\lambda}{\mu} \frac{\partial \zeta}{\partial x} + \underbrace{\mu \epsilon_0 \phi_0}_{\mathcal{D}_i} \frac{|u|u}{D} = 0$$

$$\epsilon_0 = \frac{A_0}{D_0}$$

$$\delta_0 = \frac{L_{w0}}{L_{b0}}$$

$$\phi_0 = \frac{C_{f0} L_{w0}}{D_0}$$

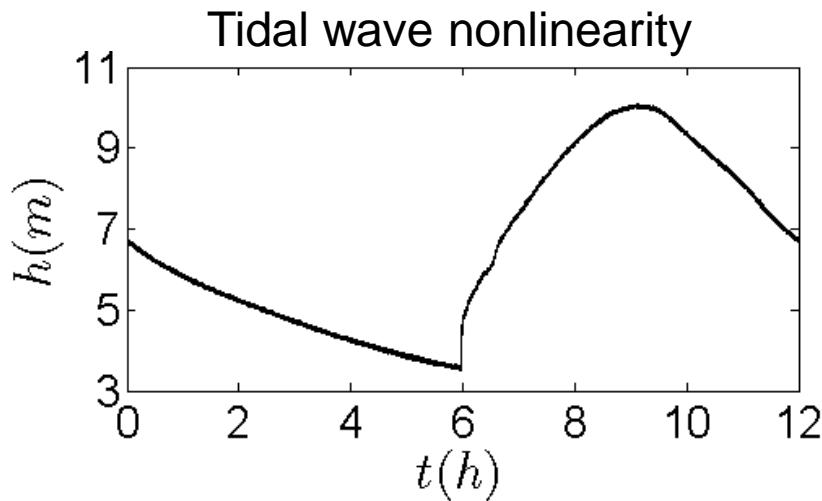
$$L_{w0} = (g D_0)^{1/2} \omega_0^{-1}$$

$$\mu = \frac{U_0}{\epsilon_0 (g D_0)^{1/2}}$$

$$\lambda = \frac{L_{w0}}{L_0}$$

$$\frac{\partial \zeta}{\partial t} + \mu\lambda \left(\epsilon_0 u \frac{\partial \zeta}{\partial x} + D \frac{\partial u}{\partial x} \right) - \mu\delta_0 u D = 0$$

$$\frac{\partial u}{\partial t} + \mu\lambda\epsilon_0 u \frac{\partial u}{\partial x} + \frac{\lambda}{\mu} \frac{\partial \zeta}{\partial x} + \underbrace{\mu\epsilon_0\phi_0}_{\mathcal{D}_i} \frac{|u|u}{D} = 0$$



$$D_i > 1.5$$

Bonneton et al., JGR 2015

→ necessary condition for tidal bore formation by not a sufficient one

$$\frac{\partial \zeta}{\partial t} + \mu \lambda \left(\epsilon_0 u \frac{\partial \zeta}{\partial x} + D \frac{\partial u}{\partial x} \right) - \mu \delta_0 u D = 0$$

$$\frac{\partial u}{\partial t} + \mu \lambda \epsilon_0 u \frac{\partial u}{\partial x} + \frac{\lambda}{\mu} \frac{\partial \zeta}{\partial x} + \underbrace{\mu \epsilon_0 \phi_0}_{\mathcal{D}_i} \frac{|u|u}{D} = 0$$

→ explore the 3D dimensionless external parameter space: $(\epsilon_0, \delta_0, \phi_0)$

- Field data: 21 convergent alluvial estuaries

Bonneton, P., Filippini, A.G., Arpaia, L., Bonneton, N. and Ricchiuto, M. 2016.

Conditions for tidal bore formation in convergent alluvial estuaries. ECSS, 172, 121-127

- Numerical simulations: 225 runs of a Serre/Green Naghdi model

Filippini, A.G., Arpaia, L., Bonneton, P., and Ricchiuto, M. 2017.

Modelling analysis of tidal bore formation in convergent estuaries. in press EJM/B Fluids

Conditions for tidal bore formation

Field data

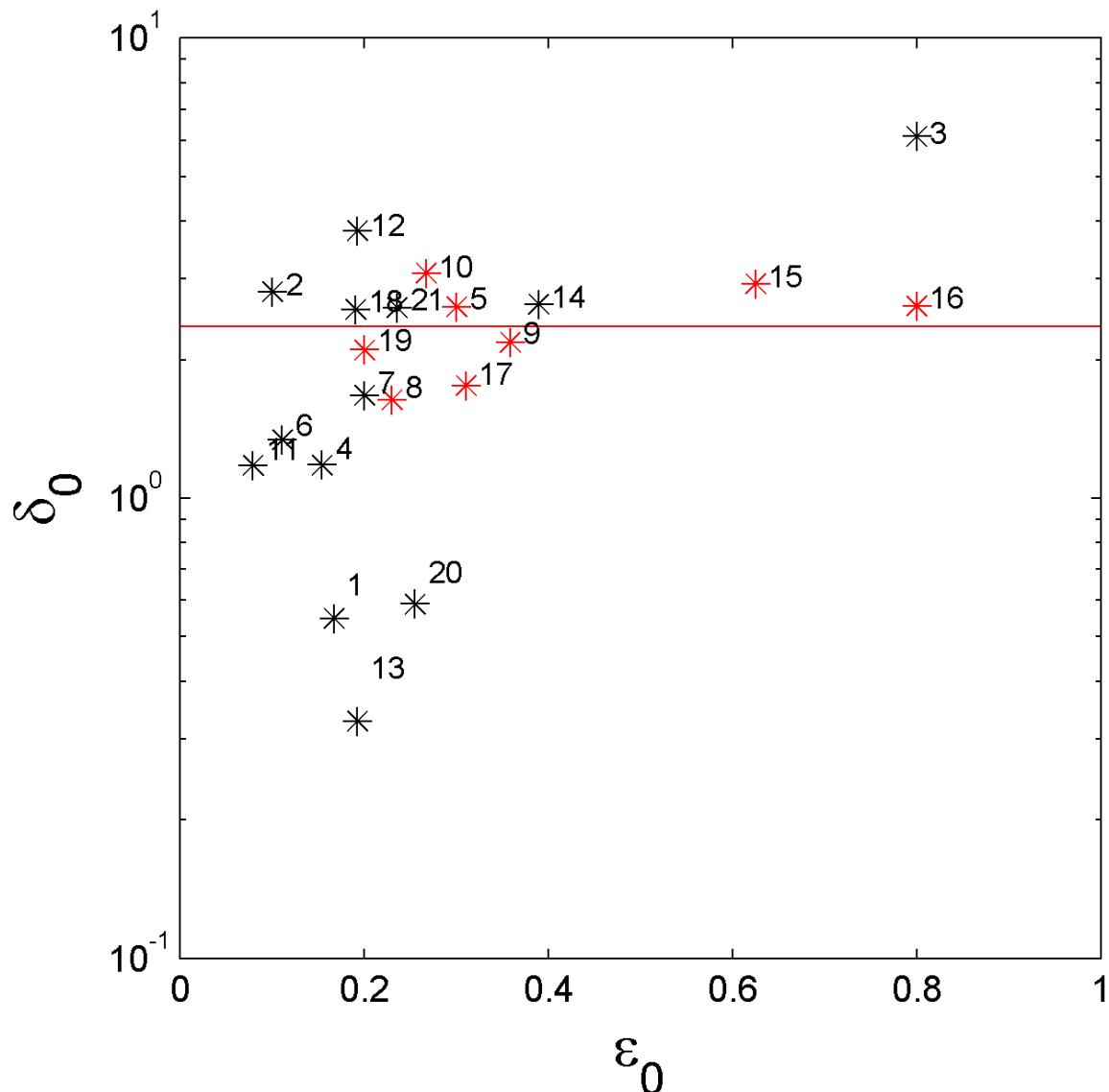
1	Chao Phya	Thailand
2	Columbia	USA
3	Conwy	UK
4	Corantijn	USA
5	Daly	Australia
6	Delaware	USA
7	Elbe	Germany
8	Gironde	France
9	Hooghly	India
10	Humber	UK
11	Limpopo	Mozambique
12	Loire	France
13	Mae Klong	Thailand
14	Maputo	Mozambique
15	Ord	Australia
16	Pungue	Mozambique
17	Qiantang	China
18	Scheldt	Netherlands
19	Severn	UK
20	Tha Chin	Thailand
21	Thames	UK

- 21 convergent alluvial estuaries
- 8 tidal bore estuaries

$$D_0, L_{b0}, A_0 = Tr_0/2, Cf_0$$

Conditions for tidal bore formation

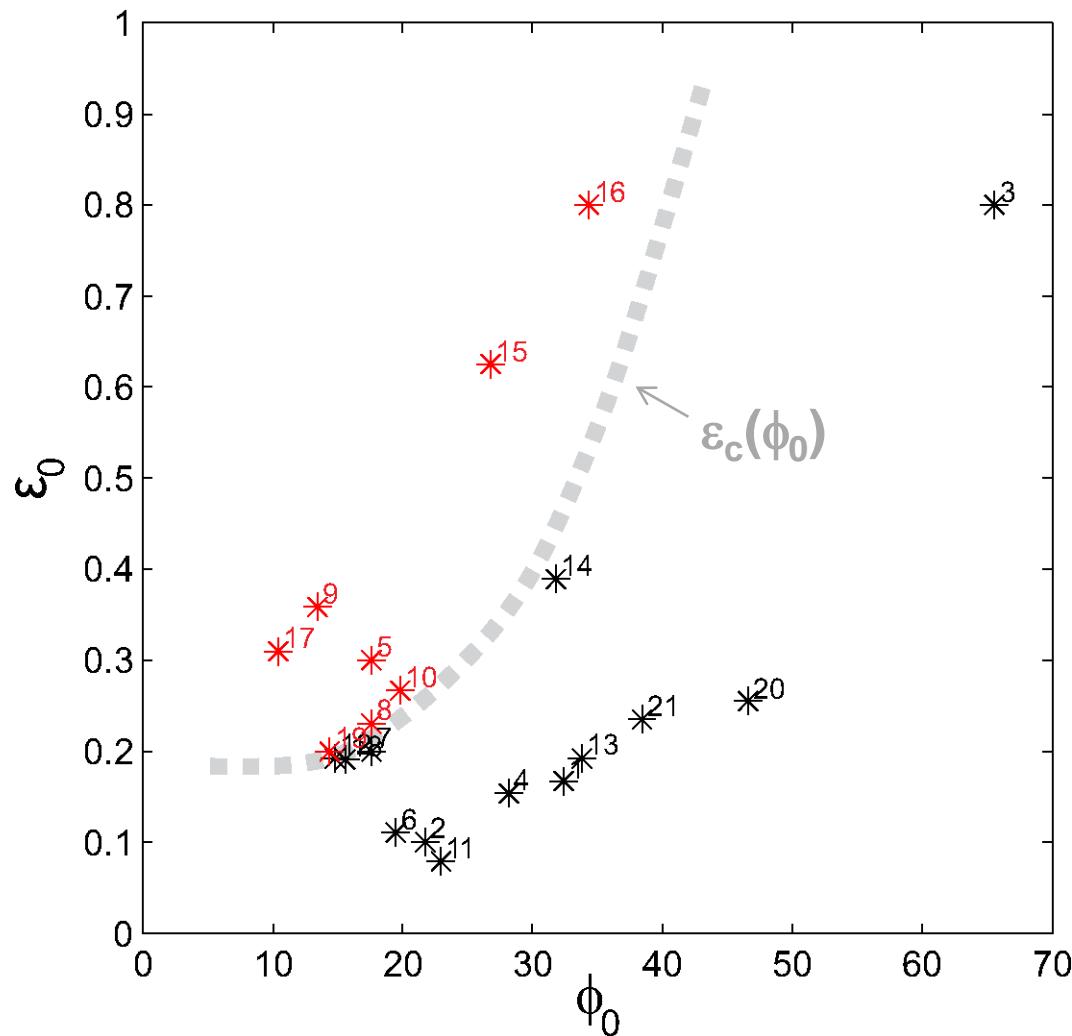
Field data



Tidal bore estuaries: $\delta_0 \approx 2.4 \rightarrow$ 2D parameter space (ϵ_0 , ϕ_0)

Conditions for tidal bore formation

Field data

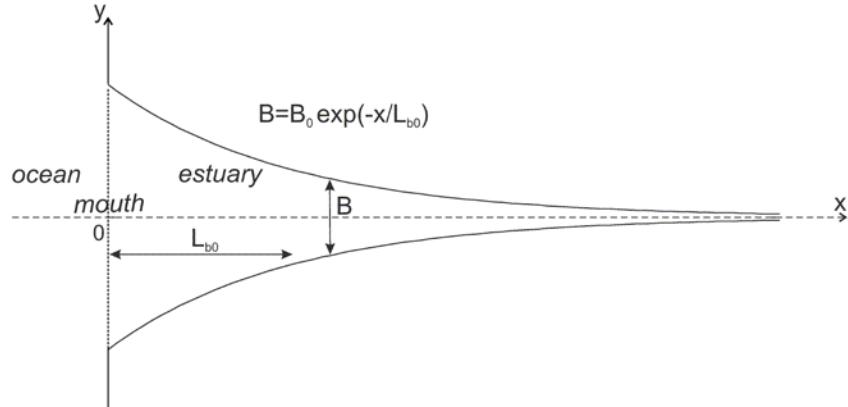


Tidal bores occur when $\varepsilon_0 > \varepsilon_c(\phi_0)$

Numerical investigation of the 2D parameter space (ε_0, ϕ_0)

Filippini, A.G., Arpaia, L., Bonneton, P., and Ricchiuto, M., EJM-B / Fluids, 2017

- Idealized convergent estuaries



- fully nonlinear weakly dispersive Boussinesq type equations:
Serre/Green Naghdi equations

$$\partial_t \zeta + \nabla \cdot (h\mathbf{u}) = 0$$

$$\partial_t (h\mathbf{u}) + \nabla \cdot (h\mathbf{u} \otimes \mathbf{u}) + gh\nabla\zeta + C_{f0}\|\mathbf{u}\|\mathbf{u} = \underbrace{(I + \alpha\tau)^{-1}[\tau(gh\nabla\zeta) - Q(\mathbf{u})]}_{\psi}$$

Numerical investigation of the 2D parameter space (ε_0, ϕ_0)

Filippini, A.G., Arpaia, L., Bonneton, P., and Ricchiuto, M., EJM-B / Fluids, 2017

Hybrid FV/FE scheme

$$\begin{aligned} \partial_t \zeta + \nabla \cdot (h\mathbf{u}) &= 0 \\ \partial_t(h\mathbf{u}) + \nabla \cdot (h\mathbf{u} \otimes \mathbf{u}) + gh\nabla\zeta + C_{f0}\|\mathbf{u}\|\mathbf{u} &= \psi \end{aligned}$$

hyperbolic step

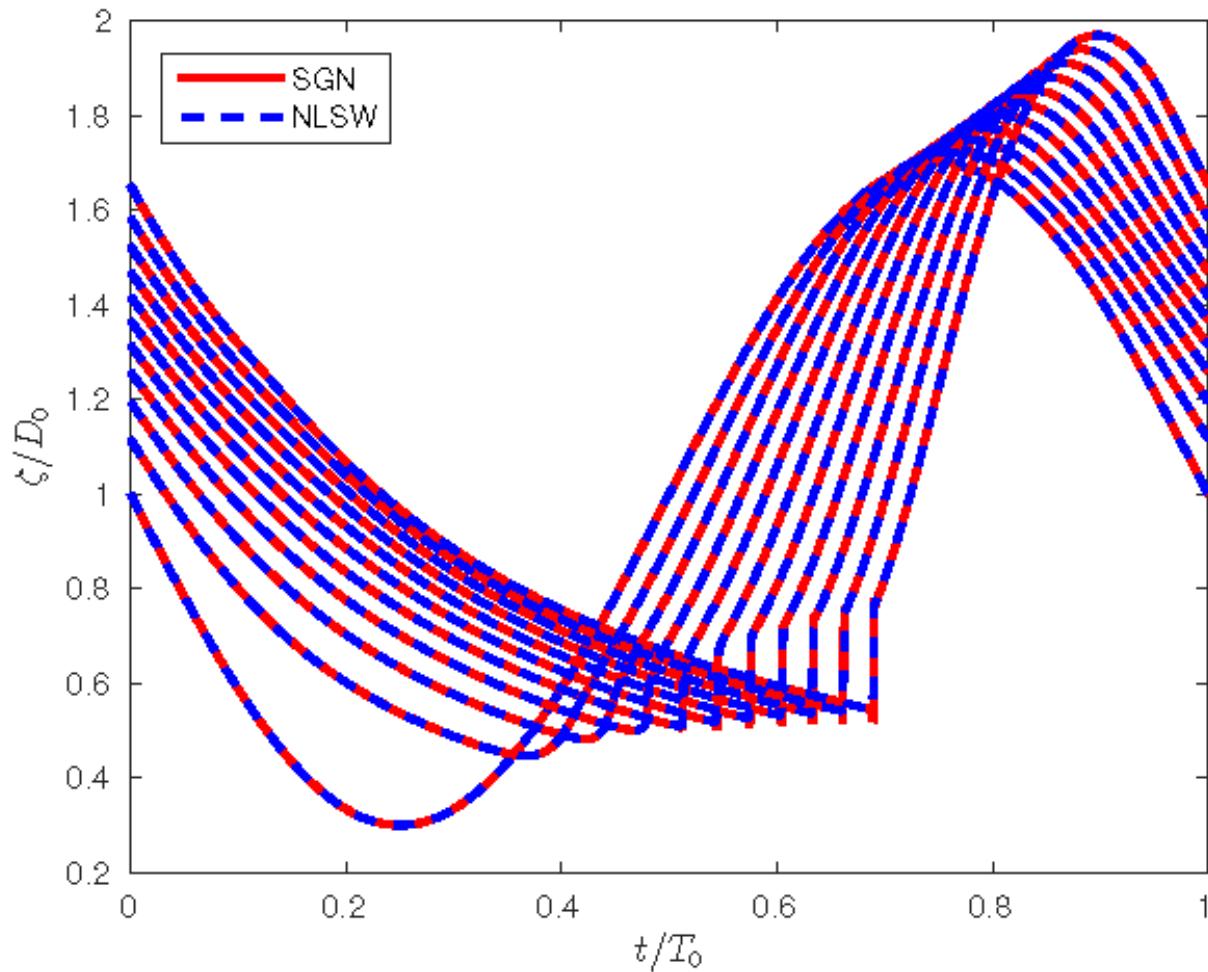
3rd order upwind MUSCL

$$(I + \alpha\tau)\psi - \tau(gh\nabla\zeta) + Q(\mathbf{u}) = 0$$

elliptic step

2nd order continuous FE

SGN / NLSW

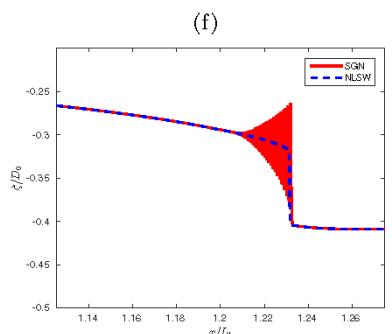
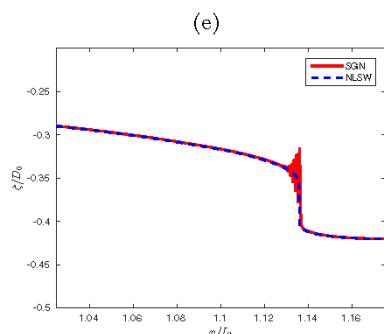
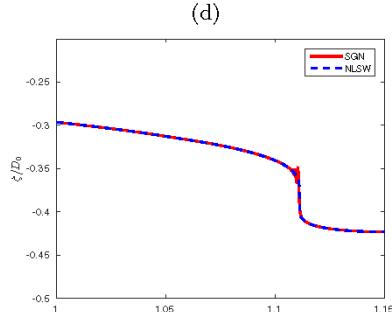
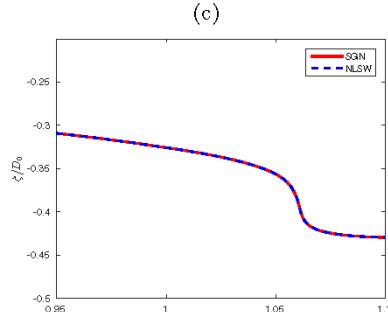
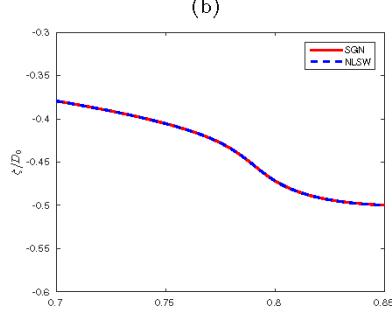
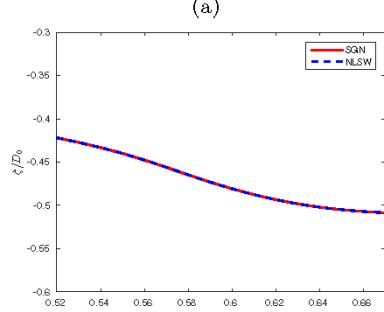
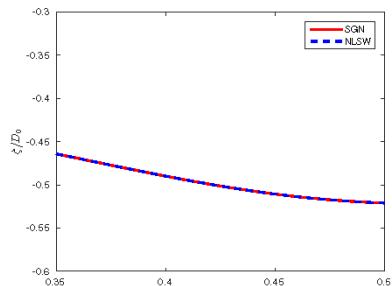
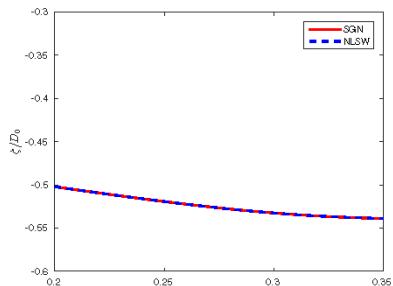


$$\varepsilon_0 = 0.7, \phi_0 = 10$$

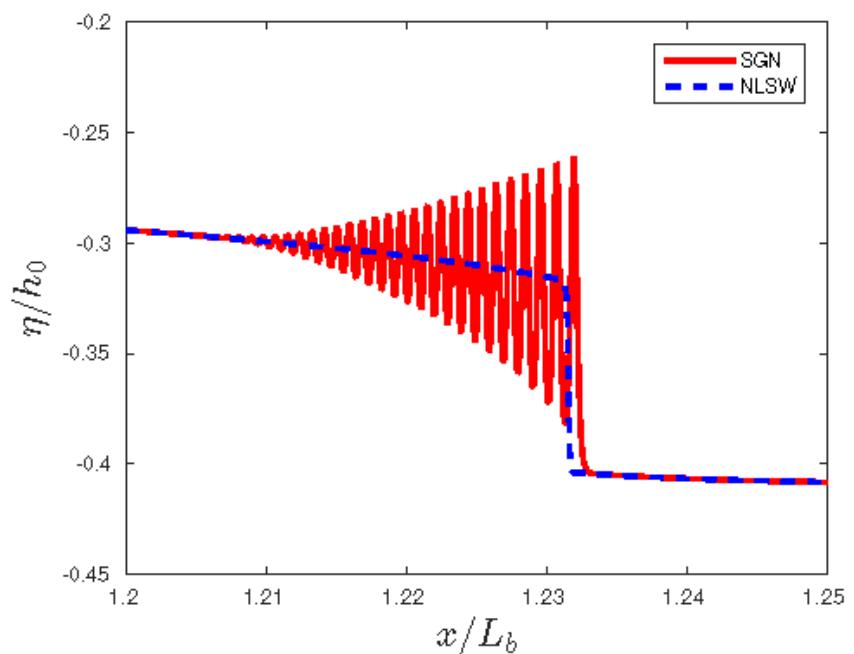
Filippini et al. 2017

Conditions for tidal bore formation

Numerical simulations



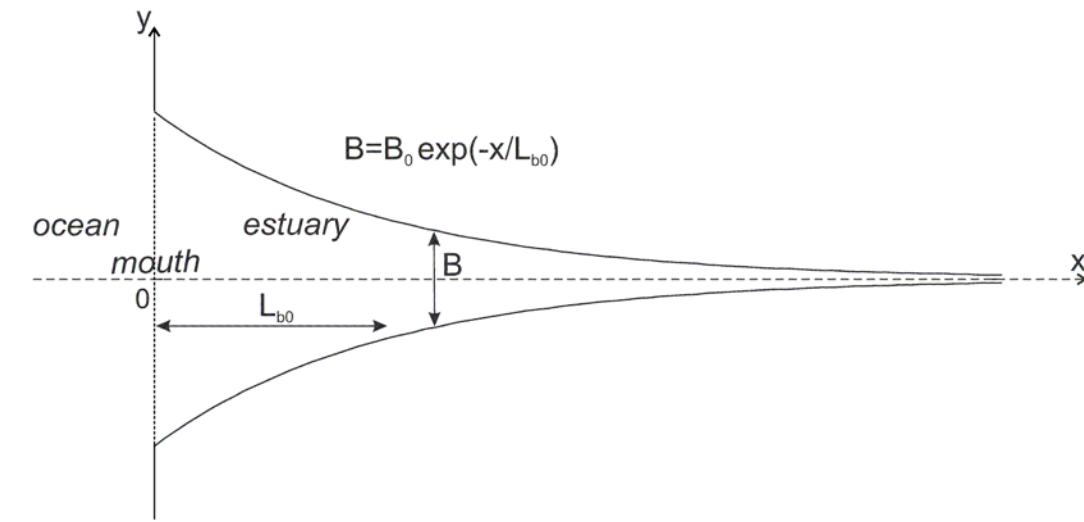
SGN / NLSW



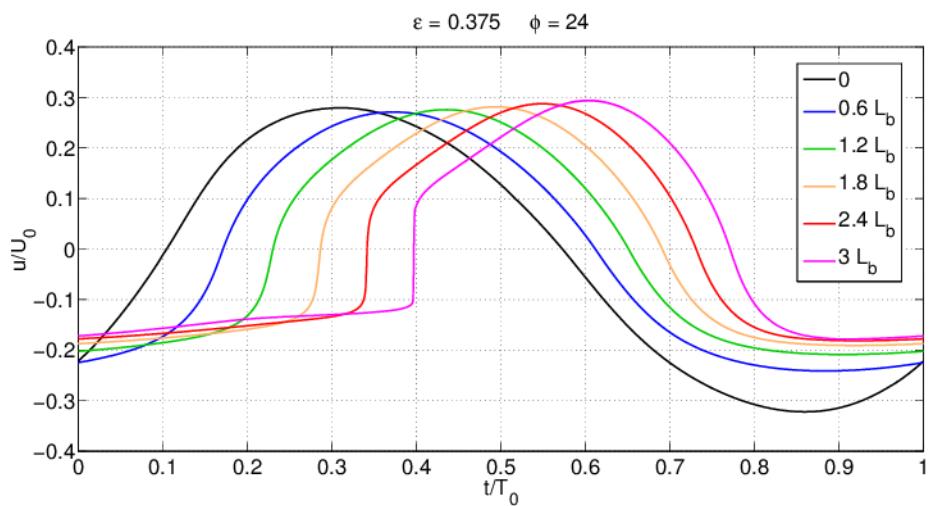
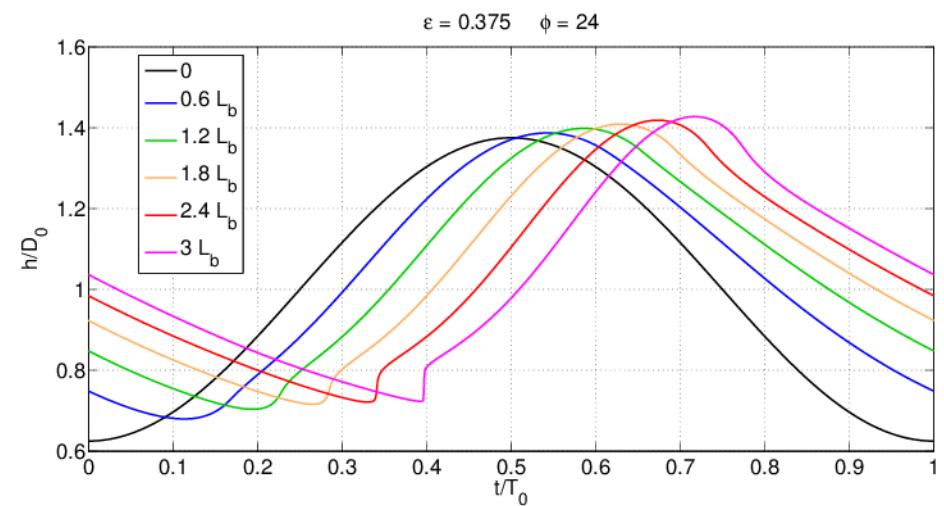
Numerical investigation of the 2D parameter space (ε_0, ϕ_0)

→ model based on the NLSW equations

→ 225 runs with $\delta_0 = 2$



one example on the 225 runs



$$S_{max} = \max \left(\frac{\partial \zeta}{\partial x} \right)$$

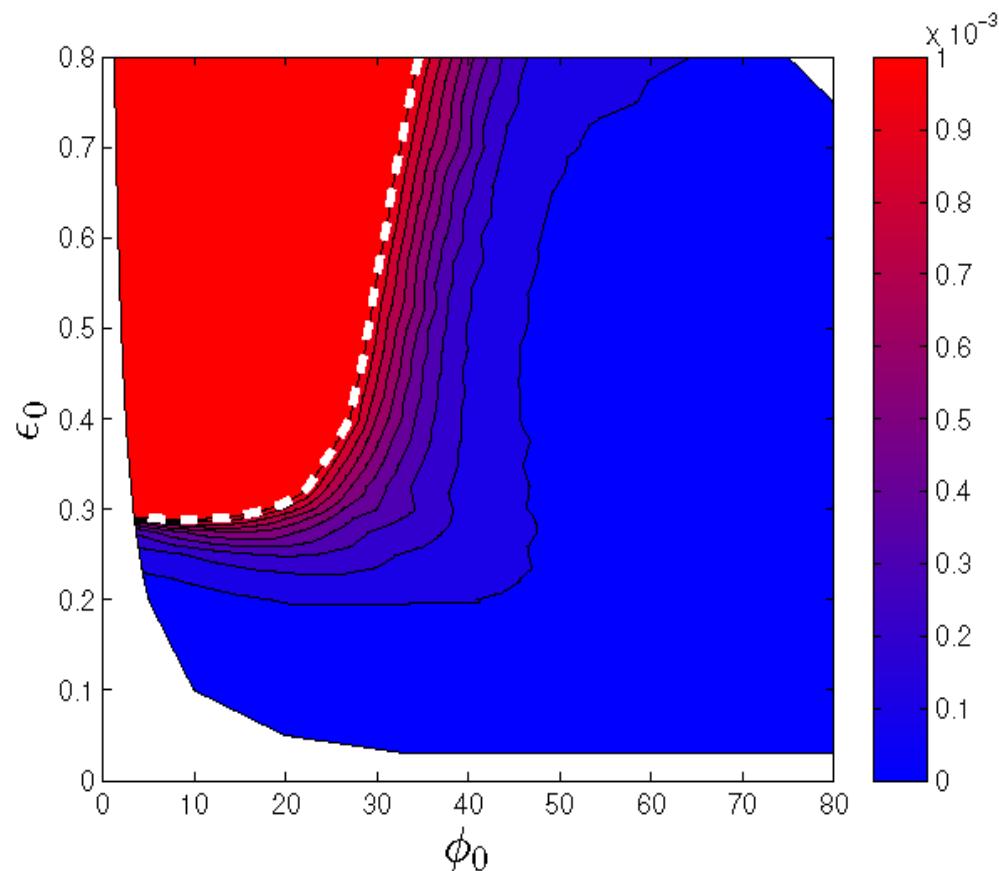
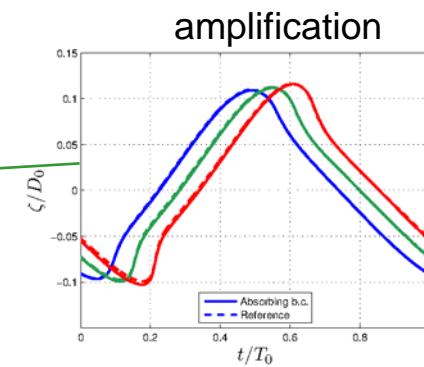
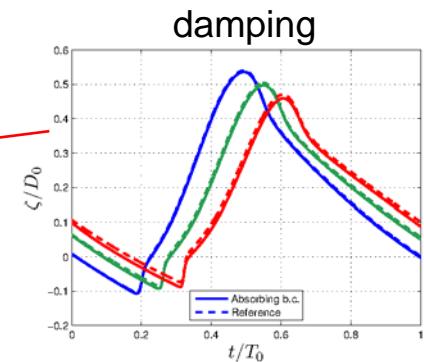
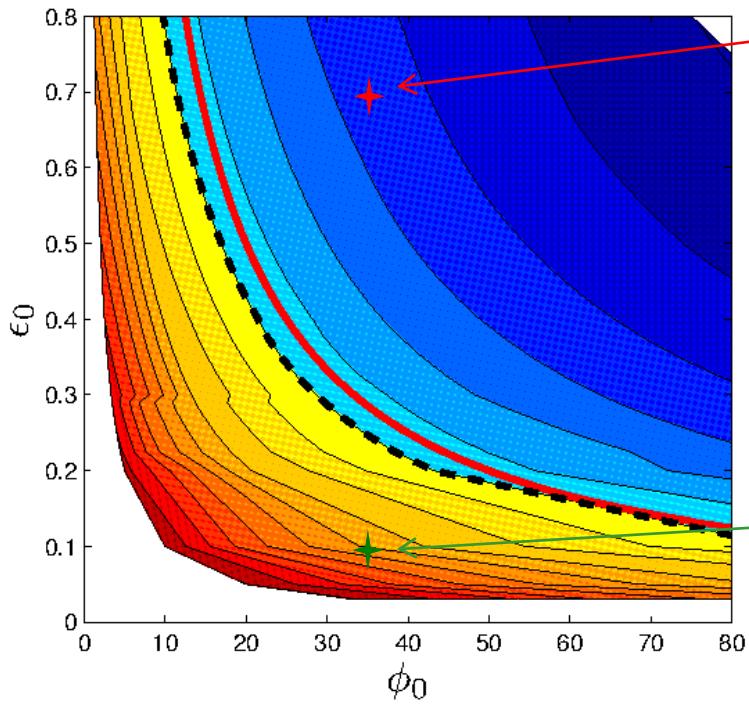


Figure 2: Isocurves of the quantity S_{max} in the plane of the parameters (ϕ_0, ϵ_0) , the white dashed line represents the $\epsilon_c(\phi_0)$ curve, namely the limit for tidal bore appearance following the criterion $S_{max} \geq 10^{-3}$.

rate of change of the tidal range

$$\Delta T_r = \frac{T_r(Lc) - T_r(0)}{T_r(0)}$$

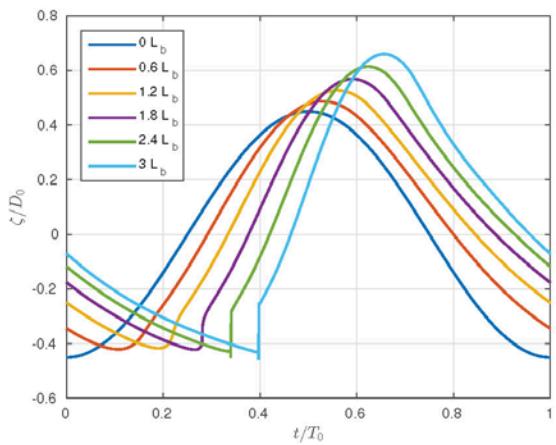


Theoretical zero-amplification curve:

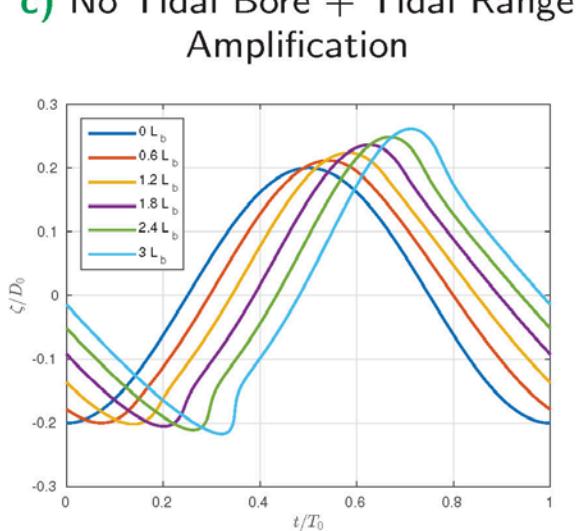
$$\varepsilon_0 \phi_0 = \delta_0 (\delta_0^2 + 1) \quad (\text{Savenije et al. 2008})$$

Conditions for tidal bore formation

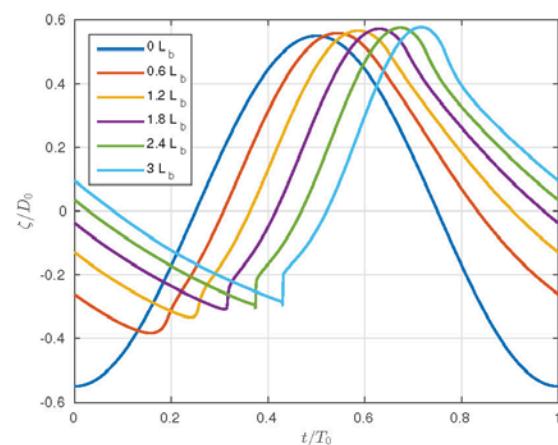
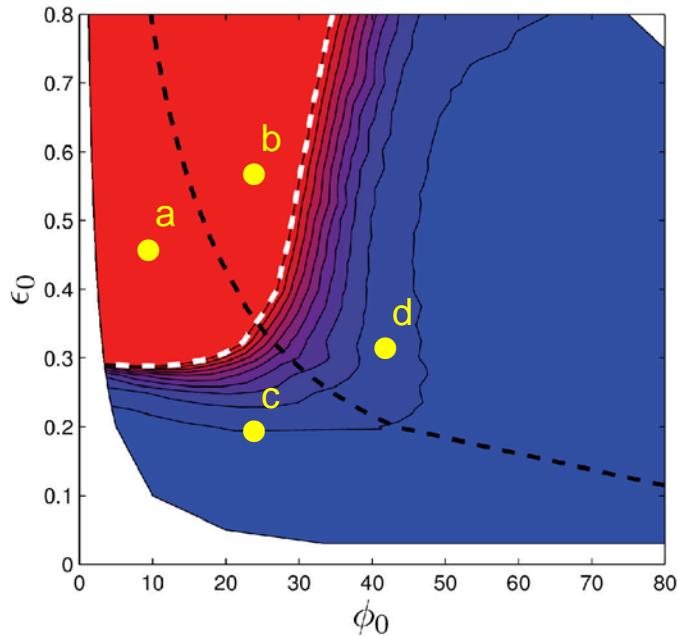
Estuary classification



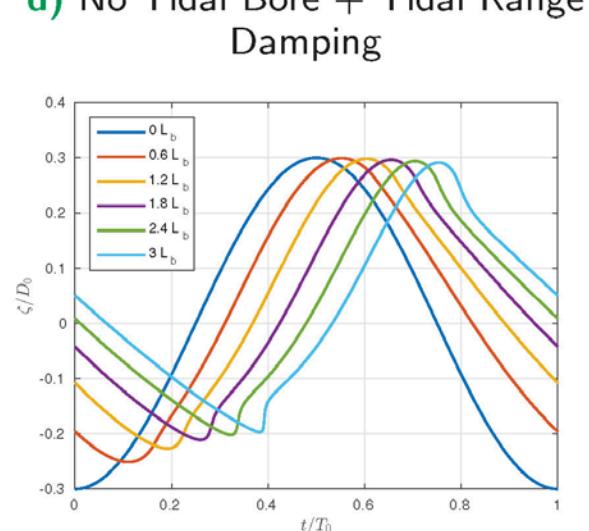
a) Tidal Bore + Tidal Range Amplification



- Marginal Curve for tide amplification

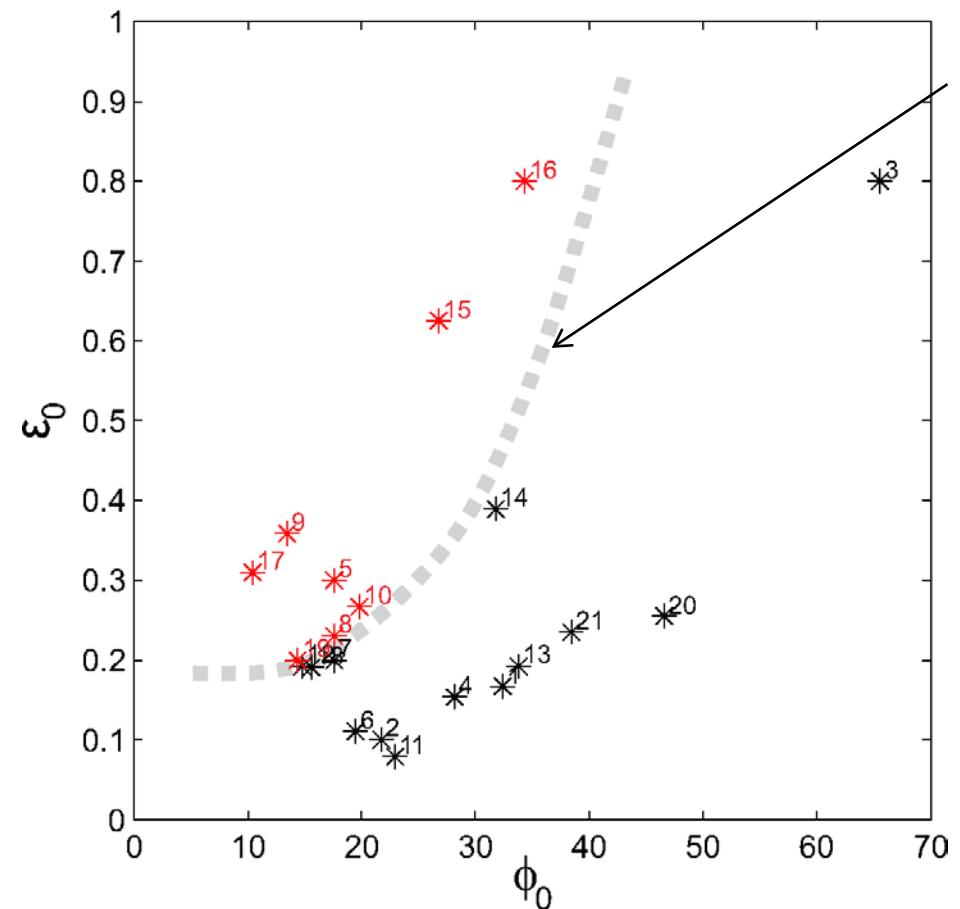


b) Tidal Bore + Tidal Range Damping

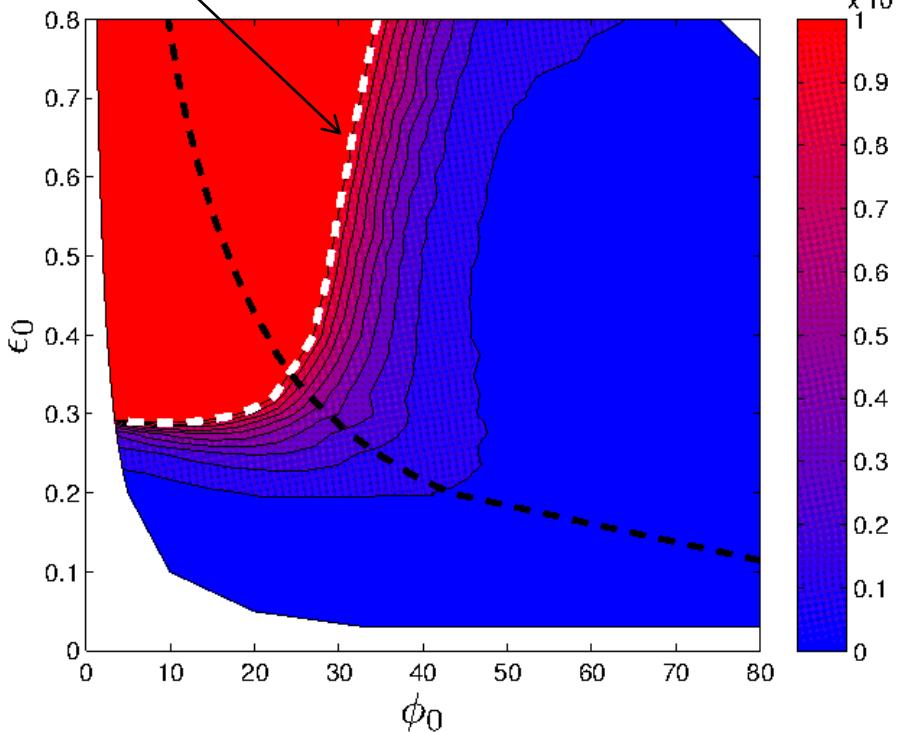


Conditions for tidal bore formation

Estuary classification



$$\varepsilon_c(\phi_0)$$



Tidal bores occur when $\varepsilon_0 > \varepsilon_c(\phi_0)$

Conclusion

Criterion for the formation of tsunami-like bore

$$\delta_0 = \frac{L_{w0}}{L_c} < \frac{3}{2}\epsilon_0$$



tsunami bore



tidal bore

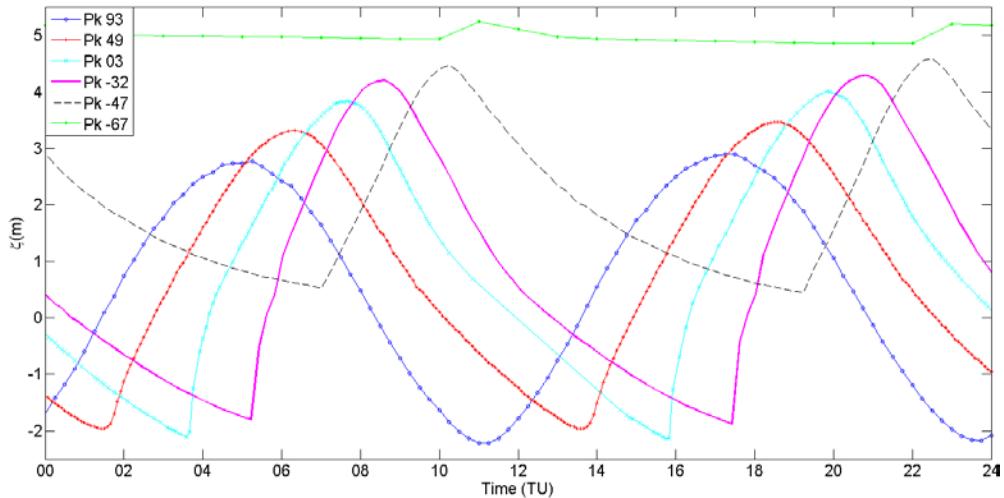
- tsunamis: bores may occur in large and shallow coastal environments
 - marine coastal plains with gentle slope (e.g.: deltas, alluvial estuaries)
 - carbonate platforms (e.g.: coral reef systems)

- tides: bores may occur in long shallow alluvial estuaries

Conclusion

Conditions for tidal bore formation

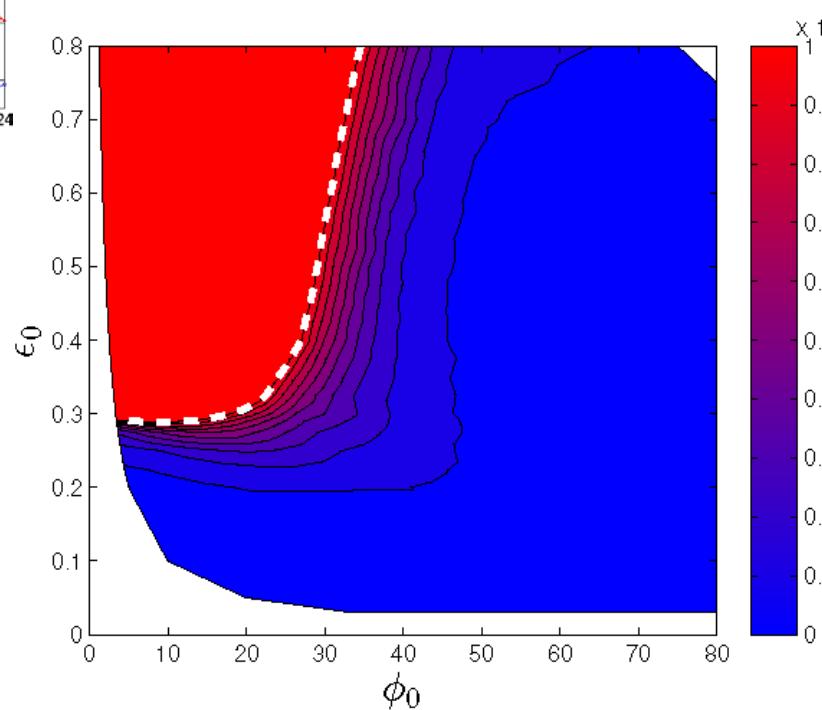
- nonlinear tidal wave transformation is mainly governed by the quadratic friction terms



$$\frac{\partial u}{\partial t} + \mu \lambda \epsilon_0 u \frac{\partial u}{\partial x} + \frac{\lambda}{\mu} \frac{\partial \zeta}{\partial x} + \underbrace{\mu \epsilon_0 \phi_0}_{\mathcal{D}_i} \frac{|u|u}{D} = 0$$

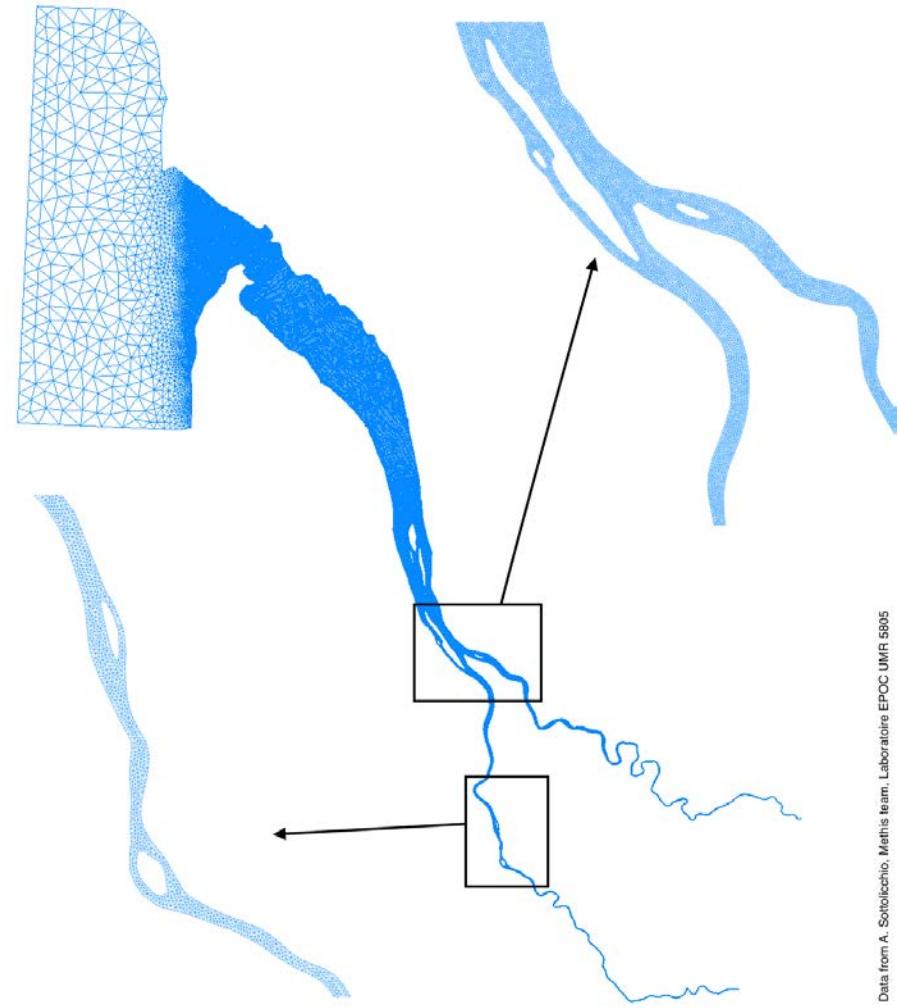
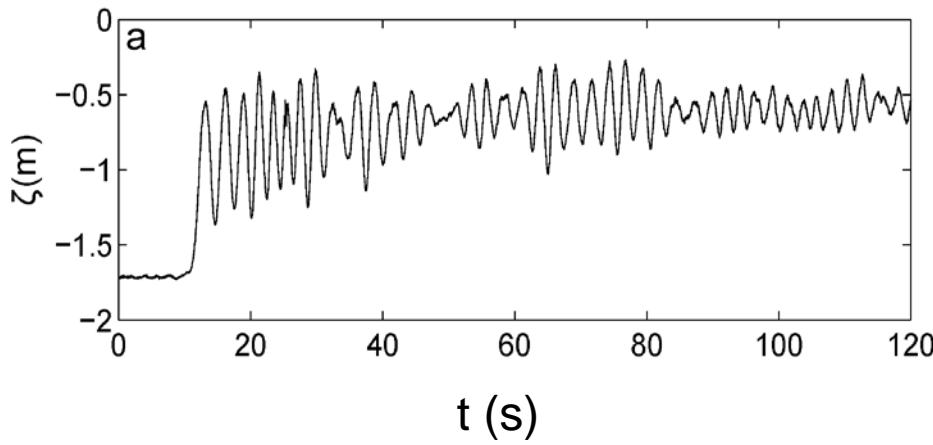
- Tidal bores occur when

$$\epsilon_0 > \epsilon_c(\phi_0)$$



Perspectives

- Modelling bore formation from large to small scales using SGN equations
→ application to tidal bores in the Gironde/Garonne estuary



Thank you for your attention

